Credit Risk Concentrations under Stress

Gabriel Bonti[1], Michael Kalkbrener[1], Christopher Lotz[2], Gerhard Stahl[2]

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Abstract

This article deals with methods for identifying as well as stressing risk concentrations in credit portfolios, in particular concentrations caused by large exposures to a single sector or to several highly correlated sectors. We present a general and yet computationally efficient framework for implementing stress scenarios in a multi-factor credit portfolio model and illustrate the proposed methodology by stressing a large investment banking portfolio. Although the methodology is developed in a particular factor model, the main concept - stressing sector concentration through a truncation of the distribution of the risk factors - is independent of the model specification. We introduce the concept of Factor Concentration that formalizes the proposed approach and analyze its mathematical properties.

1 Introduction

In a typical bank the economic as well as regulatory capital charge for credit risk far outweighs capital for any other risk class. Concentrations in a bank's credit portfolio are key drivers of credit risk capital. These risk concentrations may be caused by material concentrations of exposure to individual names as well as large exposures to a single sector (geographic region or industry) or to several highly correlated sectors. While single-name

[1]Deutsche Bank AG, Credit Risk Management, Risk Analytics & Instruments. The views expressed in this article are the authors' personal opinions and should not be construed as being endorsed by Deutsche Bank.

[2]Federal Financial Supervisory Authority (BaFin), Bonn. The views expressed in this article are the authors' personal opinions and should not be construed as being endorsed by BaFin. The third author thanks the participants of the RTF Subgroup on Concentration Risk for many useful comments and suggestions.

risk concentrations are relatively straightforward to measure and to manage, this is much harder for sector concentrations. Therefore quantitative techniques that support the identification of sector concentration are valuable tools for credit risk management. The objective of the present paper is the development of a stress testing methodology for this type of concentration risk.

The IRB approach in BIS (2005a) does not provide an appropriate quantitative framework for analyzing concentration risk. It is based on a credit portfolio model that was originally designed to produce portfolio-invariant capital charges. However, it is only applicable under the assumptions that (cf. Gordy (2003))

1. bank portfolios are perfectly fine-grained and
2. there is only a single source of systematic risk.

The simplicity of the model ensures its analytical tractability. However, it makes it impossible to model risk concentrations in a realistic way: neither name concentration is captured nor is it possible to define sector concentration in this one-factor model.

In order to develop meaningful stress tests, we need to generalize the IRB approach to a multi-factor credit portfolio model that takes into account individual exposures and has a richer correlation structure. Note, however, that in such a model concentration risk cannot be separated from credit risk. Stressing concentration risk therefore is an integral part of the stress testing methodology for credit risk.

We consider two types of scenarios for stressing sector concentration:

1. economic stress scenarios or market shocks and
2. portfolio specific worst case scenarios.

These scenario types serve different purposes. Economic stress scenarios and market shocks are usually specified by risk management. The objective is to quantify the impact of a plausible economic downturn or a market shock on a credit portfolio. This type of stress test is designed to provide information that can be easily translated into concrete management actions.

The aggregated loss of portfolio specific worst case scenarios, on the other hand, serves more as a benchmark to create some awareness of the current market situation. The construction of these scenarios is driven by portfolio characteristics instead of economic considerations.

In order to implement an economic stress scenario in the credit risk model, the model should include a set of systematic risk factors that have a clear economic interpretation, e.g. the systematic factors represent either countries or industries. Via this link, the economic stress scenario can be translated into constraints on the corresponding systematic factors. These constraints are used to truncate the distribution of the stressed risk factors or - in other words - restrict the state space of the model, where each state represents values of the systematic and idiosyncratic factors. The response of the peripheral (or unstressed) risk factors is specified by the dependence structure of the model. This approach is superior to a simple aggregation of exposures by sector, because it can also be used for the identification of risk concentrations across distinct, but highly correlated sectors. A comparable stress testing procedure is developed independently in Elsinger, Lehar and Summer (2005) where it is used in the analysis of systemic stability of a banking system.

The translation of stress scenarios into constraints on the state space of the model has a number of advantages:

1. Stress scenarios are implemented in a way that is consistent with the existing quantitative framework. This implies that the relationships between (unrestricted) risk factors remain intact and the experience gained in the day-to-day use of the model can be used to interpret the results from stress testing.

2. The probability of each stress scenario, e.g. the probability that the risk factors satisfy all the constraints under non-stress conditions, can be easily calculated. This is a good indicator for the severity of a stress scenario.

3. It is a flexible framework for the implementation of stress tests of different complexity, while at the same time being computationally efficient: importance sampling techniques can be applied to keep the computational effort close to an unconditional simulation.

Although our stress testing methodology is developed in a particular factor model, the main concept - stressing sector concentration through a truncation of the distribution of the risk factors - is completely independent of the model specification and the way that default dependencies are parameterized, e.g. whether asset or default correlations are used. In fact, it can be applied to factor models for market and operational risk as well. The proposed notion of sector (or factor) concentration is also largely independent of the marginal distributions of the risk factors and the portfolio loss, thus focusing on the dependence of these variables. It can be considered as a generalization of Tail Dependence (see Embrechts, McNeil and Straumann (2002) for a definition of this measure of dependence).

In the present paper, stress tests are performed using Deutsche Bank's proprietary credit portfolio model, which is an implementation of a Merton-type model similar to Moody's KMV model (see e.g. Crosby and Bohn (2003)) or CreditMetrics (see Gupton, Finger and Bhatia (1997)). This quantitative framework provides the necessary flexibility to incorporate stress scenarios by restricting the state space of the model. The actual calculation of the stressed loss distribution of the portfolio is done through Monte Carlo simulation on the restricted model space. It is therefore straightforward to calculate risk measures like Expected Loss, Value-at-Risk (VaR) or Expected Shortfall for the loss distribution under stress and to use statistical techniques such as QQ-plots to study its behaviour.

It depends on the particular purpose of a stress test which of those risk measures is used to quantify the impact of a stress test on the credit portfolio. One possibility is to analyse whether current capital requirements cover realized losses in stress scenarios and to use stress tests for the calculation of the conditional expected loss. Another application of stress tests is the analysis of future capital requirements, e.g. the bank wishes to satisfy its EC constraint one year into the future. If the stress event arrives within the one year horizon, then the bank will need capital sufficient to meet its EC requirement conditional on that stress event. This type of analysis requires the calculation of the VaR of the stressed portfolio. Finally, the future regulatory capital requirements in stress scenarios can be assessed by recalculating the Basel II formula with the stressed PDs from the multi-factor model. The notion of a dynamic VaR constraint has also been explored by Peura and Jokivuolle (2004) in their analysis of bank's regulatory capital adequacy.

The paper is structured in the following way: The second section introduces the quantitative framework we will work in. The third section provides a survey on stress testing methodology and gives an outline of our approach to stressing concentration risk. The actual implementation in a multi-factor credit portfolio model is described in the fourth section. Results from stressing a sample portfolio are presented. In section 5, the concept of factor concentration is formalized and its basic properties are analyzed. Section 6 concludes.

2 Concentration Risk and Credit Risk Models

To a certain degree, any real credit portfolio will contain concentrations of exposures. We differentiate between two kinds of concentrations:

Name concentration. When there are material concentrations of exposure to individual names, there will be a residual of undiversified idiosyncratic risk in the portfolio that is not captured by the IRB model. This form of credit concentration risk has been addressed via a granularity adjustment to portfolio capital, see for example Gordy (2004) or Martin and Wilde (2002). Name concentration only depends on the characteristics of individual portfolio positions. Hence, name concentration is easier to identify (and to measure) than sector concentration.

Sector concentration. Borrowers may differ in their degree of sensitivity to systematic risk, but few firms are completely indifferent to the wider economic conditions in which they operate. As a consequence, defaults of different borrowers are usually not independent. The realistic estimation of default dependence is essential for the quantification of credit risk. The most common approach to introduce default dependence into a credit portfolio model is through systematic factors, for example through factors corresponding to different sectors (geographic regions or industries). A large exposure to a single sector or to several highly correlated sectors can give rise to so-called sector concentrations.

In this paper, we will focus on stressing sector concentration. Our analysis will be performed in a typical multi-factor credit portfolio model which takes into account individual exposures and admits a general correlation structure between systematic risk factors. Note, however, that the model itself does not distinguish concentration risk from credit risk. On the contrary, name and sector concentrations are the main drivers of economic capital for credit risk, and are inextricably linked to the default risk of individual obligors. Therefore any attempt to separate concentration risk from the notion of basic portfolio credit risk cannot be naturally grounded in the multi-factor model but needs to draw on externally derived artificial criteria. As a consequence, stressing concentration risk in a multi-factor model is an integral part of a general stress testing methodology for credit risk. It is also a crucial prerequisite for successful risk management of a credit portfolio because the calculation of stress scenarios is imperative to fully quantify the impact of economic downturn scenarios or market shocks on risk concentrations.

We will now introduce a multi-factor credit portfolio model that will serve as the formal framework for the development of stress scenarios for concentration risk. In order to fix some notation, let us consider a portfolio of n loans with loss-at-default l_i. With each loan we associate a Bernoulli variable L_i that specifies the loan loss over one period:

$$\text{default: } L_i = l_i \text{ with probability } p_i,$$
$$\text{no default: } L_i = 0 \text{ with probability } 1 - p_i.$$

A prerequisite for the calculation of the portfolio loss is the specification of the dependence structure of the credit portfolio. A common way to describe dependencies between credits is the following Merton-type factor model, where loss variables L_i are linked to ability-to-pay variables Y_i:

$$\begin{aligned} L_i &:= l_i \quad \text{if} \quad Y_i \leq \Phi^{-1}(p_i) \\ L_i &:= 0 \quad \text{if} \quad Y_i > \Phi^{-1}(p_i). \end{aligned}$$

Here Φ^{-1} is the inverse of the standard Gaussian distribution function and Y_i is a standard Gaussian variable. The dependency structure is parameterized in terms of systematic factors X_i which drive the individual ability-to-pay variables:

$$Y_i = \sum_{j=1}^{m} \phi_{ij} X_j + \sqrt{1 - R_i^2} Z_i \text{ and} \tag{1}$$

$$Z_i \overset{iid}{\sim} \mathcal{N}(0, 1), \tag{2}$$

where $0 \leq R_i^2 \leq 1$ and $(\phi_{i1}, \ldots, \phi_{im})$ is a weight vector. The systematic factors form an m-dimensional Gaussian vector $(X_1, \ldots, X_m)$ with mean 0 and covariance matrix Σ. For the portfolio $L = \sum L_i$, we can now calculate its Expected Loss $\mathbb{E}(L)$, Value-at-Risk $\mathrm{VaR}_\alpha(L)$ at level $\alpha \in (0, 1)$ defined as an $\alpha-$quantile of L, and Economic Capital $\mathrm{EC}(L)$:

$$\begin{aligned}
\mathbb{E}(L) &:= \sum l_i p_i, \\
\mathrm{VaR}_\alpha(L) &:= \inf\{x \in \mathbb{R} \mid \mathbb{P}(L \leq x) \geq \alpha\}, \\
\mathrm{EC}(L) &:= \mathrm{VaR}_\alpha(L) - \mathbb{E}(L).
\end{aligned}$$

Seen in the context of a multi-factor credit portfolio model, sector concentration risk is due to the systematic risk factors driving individual credits' asset price processes. For the rest of this paper, we will place ourselves in the setting of this model. We want to emphasize, however, that the stress testing methodology presented in this paper is applicable to a much wider class of models including credit portfolio models with non-Gaussian risk factors and macroeconometric models as presented in Pesaran et al. (2004, 2005, 2006).

3 Stress Scenarios

Stress testing has been adopted as a generic term describing various techniques used by financial firms to gauge their potential vulnerability to exceptional but plausible events (see BIS (2000, 2001, 2005b) and Blaschke, Jones, Majnoni, and Martinez Peria (2001) for industry studies on stress testing and Lopez (2005) for an overview of stress testing in banking supervision). The most common of these techniques involve the determination of the impact on the portfolio of a bank or business unit of a move in a particular risk factor (a simple sensitivity test) or of a simultaneous move in a number of risk factors, reflecting an event which the bank's risk managers believe may occur in the foreseeable future (scenario analysis).[3] The following classification should serve as a rough guide and distinguish different types of stress scenarios.

1. *Macroeconomic scenarios.* A macroeconomic scenario usually requires the use of a macroeconomic model. It specifies an exogenous shock to the whole economy that is propagated over time and may impact the banking system in various ways. This type of stress scenario is sometimes used by financial regulators or central banks in order to gain an understanding of the resilience of financial markets or the banking system as a whole, see for example DeBandt and Oung (2004).

2. *Market shocks.* These scenarios specify shocks to financial markets. This category also includes certain shocks of a "systemic" nature affecting credit risk (such as a sudden flight to liquidity), or sectoral shocks, for instance the deterioration in credit spreads in the TMT (Technology Media-Telecommunications) sector. Historical scenarios are frequently used for this type of shocks in order to increase the plausibility of these stress scenarios.

3. *Portfolio specific worst case scenarios.* The objective of this worst case analysis is to identify scenarios that are most adverse for a given portfolio (Breuer and Krenn (2000)). The specification of worst case scenarios can either be based on expert judgement or quantitative techniques, for instance importance sampling (see, for example, Kalkbrener, Lotter and Overbeck (2004)). The aggregated loss in these scenarios serves as a benchmark to create some awareness of the current market situation.

Stress scenarios are typically analyzed within the existing model. The focus is different for those tests where the model itself is challenged, and alternative assumptions or models are used to value a portfolio or measure its risk. This implies that the usual framework for risk management is abandoned, and the experience gained in the old framework may no longer be valid in the alternative model. As a consequence, model stress is not part of

[3] Extreme Value Theory (EVT) is another technique used by some banks to capture their exposure to extreme market events. We refer to Embrechts, Klüppelberg, and Mikosch (1997) for an introduction to Extreme Value Theory and to Longin (2000) and Schachter (2001) for an application of EVT to stress testing.

the day-to-day risk management process in a financial institution. It is typically used to analyze the sensitivity of model outputs with respect to specific model assumptions and therefore a way to gauge model risk.

Regardless of the motivation for considering a particular scenario, there exist a number of criteria that characterize useful stress scenarios:

1. *Plausible.* Stress scenarios must be realistic, e.g. have a certain probability of actually occurring. Risk management will not take any actions based on scenarios that are regarded as implausible.

2. *Consistent.* One objective is to implement stress scenarios in a way that is consistent with the existing quantitative framework. This has the advantage that the relationships between risk factors remain intact and the experience gained in the day-to-day use of the model can be used to interpret scenario results.

3. *Adapted.* Stress tests should include scenarios that are specifically designed for the portfolio at hand. They should reflect certain portfolio characteristics and particular concerns in order to give a complete picture of the risks inherent in the portfolio.

4. *Reportable.* Stress scenarios should provide useful information for risk management purposes which can be translated into concrete actions. For reporting purposes, it is crucial that the stress scenario is characterized by a clearly identifiable set of stressed risk factors, sometimes called the "core" factors. The remaining "peripheral" factors should then move in a consistent way with those "core" factors.

In many banks stress scenarios supplement statistical VaR models in order to improve the risk assessment under exceptional circumstances. For integrating scenarios and statistical models, Cherubini and Della Lunga (1999) use Bayesian statistics whereas Berkowitz (1999) proposes the application of a mixture model. Both integration techniques require knowledge of the precision or probability of a stress scenario. For many hypothetical as well as historical scenarios, however, the estimation of their likelihood in the future is very difficult. In fact, it is recognized in the finance industry that the lack of probability measures for scenarios is the main limitation for their application in a quantitative framework (see BIS

(2000)). It is an important feature of our approach that the probability of a stress scenario in the existing model can be easily calculated (see section 4.2).

In the general multi-factor framework (1), stress scenarios for sector concentration apply stress to the systematic factors of the model. When designing specific stress scenarios, we usually focus on a small number of directly stressed factors, e.g. those factors that correspond to the sectors of interest. In addition, a small number of stressed factors makes it easier to transform the stress results into concrete management actions. The response of the other risk factors is specified by the dependence structure of the model (see also Kupiec (1998)). This approach is also a superior way to identify risk concentrations compared to just aggregating exposures per sector, because there it can happen that concentrations in distinct but highly correlated sectors remain undetected.

In order to increase plausibility and relevance of individual stress scenarios

- we derive the stress applied to systematic factors of the credit risk model from economic (or market) stress scenarios or
- use quantitative techniques to identify those systematic factors with the highest weights in a given portfolio (or combinations of these factors.)

In summary, we propose the following stress test for sector concentration:

1. Specify economic stress scenario or scenario based on the characteristics of the portfolio
2. Translate scenario into stress of systematic factors of credit risk model
   The stress scenarios are chosen in such a way that the translation involves only a small number of systematic factors. All other factors are impacted through correlations to the stressed factors. In this way a consistent set of stressed PDs is generated for all credits in the portfolio, where the change in PD depends on the credit's correlation to the stressed factors.
3. Determine impact of stress scenario by calculating conditional expected loss and other statistics of the portfolio.

4 Factor Stress Methodology

The objective of this section is to describe how an economic stress scenario for sector concentration can be implemented in a credit portfolio model. In this paper, Deutsche Bank's model serves as quantitative framework. Because the proposed stress testing methodology is independent of the exact model specification, our presentation of the model is focused on its relevant features.

4.1 Interpretation of systematic factors in the portfolio model

First of all, a precise meaning has to be given to the systematic factors in (1). Recall that each ability-to-pay variable

$$Y_i = \sum_{j=1}^{m} \phi_{ij} X_j + \sqrt{1 - R_i^2} Z_i$$

is a weighted sum of m systematic factors $X_1, \ldots, X_m$ and one specific factor Z_i. The systematic factors[4] correspond either to countries or industries. In our model 75 systematic factors are used, for example factors for Germany, U.K, South America, the automotive and the electrical engineering industry, etc. The systematic weights ϕ_{ij} are chosen according to the relative importance of the corresponding factors for the given counterparty, e.g. the automobile company BMW might have the following representation:

$$\begin{aligned} \text{BMW assets} &= 0.8 \times \text{German factor} + 0.2 \times \text{US factor} \\ &+ 0.9 \times \text{Automotive factor} + 0.1 \times \text{Finance factor} \\ &+ \text{BMW's non-systematic risk.} \end{aligned}$$

The specific factor is assumed independent of the systematic factors. Its role is to model the remaining (non-systematic) risk of the counterparty.

The joint probability distribution of the systematic factors is assumed Gaussian with zero mean. To measure credit risk, country and industry factors are simulated together with the specific risk factors.

[4] As opposed to specific risk, these factors influence the default of more than one obligor and therefore introduce a correlated default structure.

4.2 Specification and implementation of stress scenarios

Multi-factor credit risk models offer a number of possibilities to implement stress tests, e.g. adjustments of model parameters or distributions of risk factors, etc. The basic idea in our stress testing approach is the specification of stress scenarios as constraints on systematic risk factors. More precisely, these constraints are used to restrict the sample space in the Monte Carlo simulation of the model. This is a very general framework for stress testing. It offers the additional advantage that we can estimate the probability of each stress scenario, e.g. the probability that the risk factors satisfy all the constraints under non-stress conditions. This is a good indicator for the severity of a stress scenario.

In the following, we will describe our approach by means of a specific scenario. As an example, consider a downturn scenario for the automotive industry. The simplest implementation in the portfolio model is the following restriction of the state space of the model: only those samples are considered in the Monte Carlo simulation where the automotive industry factor decreases by a certain percentage, say at least 2%. In other words, the distribution of the automotive industry factor is truncated from above at -2%. More precisely, the steps in the calculation of stressed EL and EC are:

- simulate risk factors under their original (non-stress) joint distribution
- dismiss any simulation not satisfying the scenario constraints
- derive EL, EC and other statistics from the loss distribution specified by the MC scenarios that satisfy the constraints

Note that the automotive downturn scenario does not only have an impact on the automotive industry factor: because of correlations, other country factors as well as industry factors are also affected. Figure 1 shows the stressed distribution of the automotive industry factor (left) and the impact on the factor for the chemical industry (right): the distribution of the automobile factor has been truncated, while the distribution of the chemical industry factor is no longer centered but has moved to the left.[5]

[5]The distributions in figure 1 can be represented in a simple way: if $F_{auto}(x)$ denotes the (Gaussian) distribution of the automobile factor, its truncated distribution is given by $F_{auto}(x)/F_{auto}(-2\%)$ for $x \leq -2\%$.

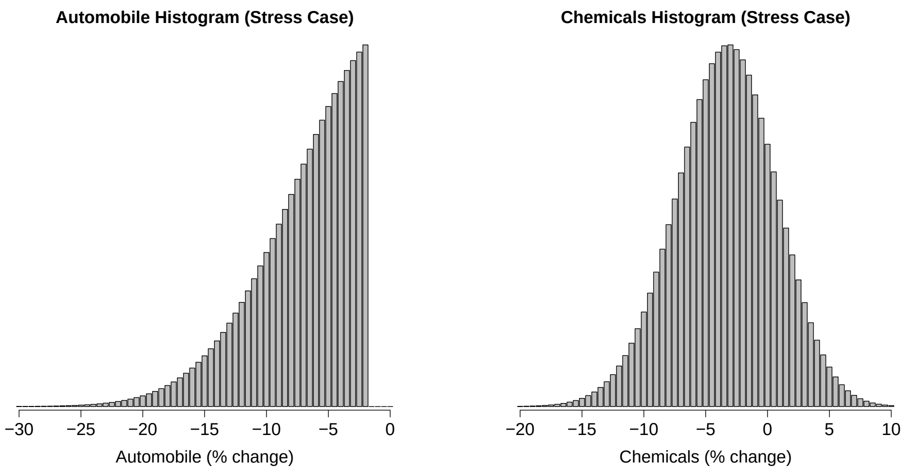

Figure 1: Histogram of simulated factor changes (stress case)

In our model the joint distribution of systematic factors is derived from stock indices and it is therefore straightforward to implement scenarios involving constraints on those indices. Sometimes, however, it is desirable to define scenarios based on other economic variables. For example, a scenario might be defined by a constraint on the production index of a specific industry. Because of the multivariate normal distribution of the factor model, correlation is the easiest way to incorporate dependence of production indices. Suppose, however, that the production index and the corresponding industry risk factor do not show a high correlation, perhaps because their relationship is nonlinear. A crude way to overcome this weak dependence is to define a scenario just in terms of the risk factor for the same industry in such a way that the probabilities of both scenarios (under non-stress conditions) agree.

In the example above, the factor stress was derived from an economic downturn scenario. An alternative approach is the specification of scenarios that are most adverse for a given portfolio. Importance sampling is a quantitative technique that was developed to improve the efficiency of Monte Carlo simulation but can also be used for the identification of worst case scenarios. We refer to Glasserman and Li (2003), Kalkbrener, Lotter and Overbeck

The factor for the chemical industry is called an incidentally truncated variable. Its marginal distribution is given by $F_{auto,chem}(-2\%, y)/F_{auto}(-2\%)$, where $F_{auto,chem}$ denotes the joint distribution of the two industry factors.

(2004) and Egloff, Leippold, Jöhri and Dalbert (2005) for importance sampling techniques in Gaussian credit portfolio models, e.g. models of the form (1). In these papers, a vector $(x_1, \ldots, x_m) \in \mathbb{R}^m$ is constructed such that a shift of the systematic factors by $(x_1, \ldots, x_m)$ reduces the variance of Monte Carlo estimates of Value-at-Risk or Expected Shortfall. The variance reduction is achieved because the systematic shift generates a large number of high portfolio losses. Hence, $(x_1, \ldots, x_m)$ is a natural candidate for the identification of those systematic factors that have to be constrained in worst case scenarios.

Restricting the state space is a flexible technique to incorporate stress scenarios into the portfolio model. Complex stress scenarios can be implemented

1. by specifying constraints that involve more than one systematic factor or
2. by defining more complex constraints than simple caps on individual factors.

One possibility is to restrict the state space of the model in such a way that the dependence of particular risk factors is increased. This technique provides an interesting alternative to simply changing correlation parameters of the model. By keeping the original model parameters intact, consistency problems are avoided such as maintaining the positive semi-definiteness of the correlation matrix of the systematic factors. We would like to emphasize that changing correlation parameters is a useful way to gauge the sensitivity of a credit risk model against errors or changes in correlation estimates (see, for instance, Kim and Finger (2000)). However, this is more a model risk exercise than a stress test since consistency with the existing quantitative framework is abandoned.

Finally, our methodology can be used to forecast regulatory capital requirements in stress scenarios: the impact of a stress scenario on regulatory capital can be assessed by re-calculating the Basel II formula with the stressed PDs from the multi-factor model. Since regulatory capital requirements are essential for capital management and strategic planning this impact analysis will be an important component of the stress testing methodology in a financial institution.

4.3 Credit risk concentrations under stress: a case study

Macroeconomic downturn scenarios are the starting point for many stress tests calculated at Deutsche Bank. These scenarios typically have a probability of 25%. Their impact on industry sectors is then analyzed and translated into forecasts for sector-specific indices. For example, consider the following downturn scenario for the automotive industry: the industry production is forecast to drop by 8% during next year. The representation of this scenario in Deutsche Bank's credit portfolio model involves the following steps. Firstly, the distribution of production index values is truncated from above, and the cut-off point is chosen so that the truncated mean equals the stress forecast. In other words, when index values are constrained to the interval below the cut-off point their mean coincides with the forecast. The cut-off point C determines the probability α of the stress scenario, i.e. α is defined such that C is the α-quantile of the distribution of index values. To represent this scenario in the credit portfolio model, an interval is chosen for the corresponding risk factor which has the same probability α, although due to the different distributions the cut-off point might not be the same.

In this case study, the stress is applied to a sample investment banking portfolio which consists of 25000 loans with an inhomogeneous exposure and default probability distribution. Its total exposure is 1000 mn EUR, average exposure size is 0.004% of the total exposure and the standard deviation of the exposure size is 0.026%. Default probabilities vary between 0.02% and 27%. Figure 2 exhibits the portfolio's exposure by rating class both for automotive companies and all other borrowers.

Application of the downturn scenario yields the following risk estimates:

	Non-stress	Stress	% chg.
Expected Loss	7.03	10.94	55.6
99.98% quantile	103.23	122.80	19.0
Cond. tail expectation at 99.98%	119.68	145.45	21.5

Table 1: Portfolio risk estimates

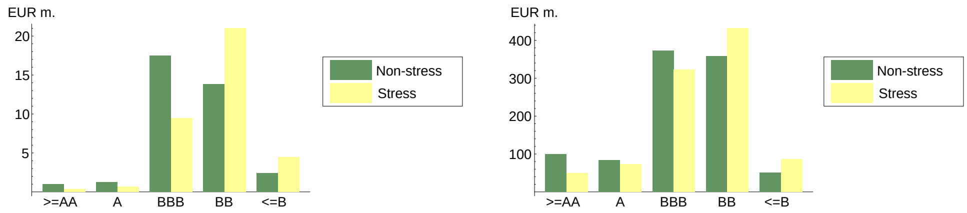

Figure 2: Exposure by rating class for automotive companies (left) and all other borrowers (right)

These key statistics provide important information on the impact of the stress scenario. For obtaining the full picture, the entire loss distribution has to be analyzed (see figure 3).

In Deutsche Bank, economic capital is based on the 99.98% quantile of the unstressed loss distribution. This quantile is also calculated in stress tests. The main motivation is the quantification of economic capital requirements in stress scenarios. For example, this case study quantifies the increase of economic capital under the specified downturn scenario.

The impact of the stress scenario on the portfolio VaR could be interpreted as a measure of "concentration" of the portfolio in the respective risk factor. However, using VaR in this way results in several problems, such as the fact that it is not indifferent to an overall change in PDs. In the next section, we will propose a superior quantity as a promising candidate to measure factor concentration.

Figure 2 exhibits the portfolio's exposure by rating class both in the non-stress and stress case. The analysis is done separately for automotive companies and all other borrowers. Figure 2 clearly shows that exposure is shifted from investments grades (BBB or above) to non-investment grades. As expected, the deterioration of ratings is more pronounced for the automotive industry. Note, however, that due to the dependence structure of the portfolio this stress scenario also has a significant impact on other borrowers.

Rather than just looking at certain quantiles or other summary statistics, we can get a better understanding of the impact of a stress scenario by studying the whole loss distribution before and after the stress. In order to see the effect of the automotive stress scenario on the portfolio loss, the left graph of figure 3 shows the original (circles) and the stressed

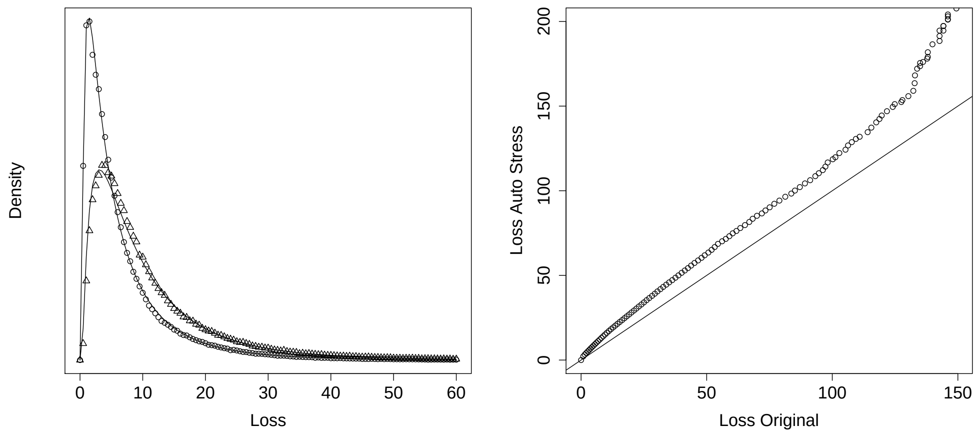

Figure 3: Left graph: Density plots of original (circles) and stressed (triangles) loss distributions, together with fitted Vasicek curves. Right graph: QQ plot of original against stressed loss distribution.

(triangles) loss densities, together with fitted Vasicek distributions (curves). By plotting quantiles of the two distributions against each other, we can clearly see where the stress scenario affects the loss distribution: while quantiles have moved higher overall, the impact is especially severe in the extreme tail of the distribution (right hand side).

The final step in this case study is the calculation of the regulatory capital requirements conditional on the stress event: recalculating the Basel II formula with the stressed PDs increases the regulatory capital from 131.41mn to 156.48mn. The increase of 19% is in line with the increase of the 99.98% quantile (see table 1).

5 A General Framework for Measuring Factor Concentration

Our approach to stressing risk concentrations is based on constraints of the form $\{X_j \leq x\}$ applied to the systematic factors $X_1, \ldots, X_m$ of the model. The objective of this section is a mathematical formalization of this concept that is independent of the specification of a particular credit risk model.

5.1 A formal definition of factor concentration

In section 4, the tail

$$1 - \mathbb{P}(L < y) = \mathbb{P}(L \geq y)$$

of the loss distribution L has been analyzed in stress scenarios of the form $\{X_j \leq x\}$. For assessing the severity of the shock and the confidence level of the tail it is convenient to express the parameters x and y in quantile space, e.g.

$$x = F_{X_j}^{-1}(p) \text{ and } y = F_L^{-1}(1-q).$$

This parametrization leads to the following formalization of *Factor Concentration*: for a given loss distribution L, the risk concentration in the systematic factor X is a function from $[0,1]^2$ to $[0,1]$ defined by

$$FC_X(p,q) := \mathbb{P}(L \geq F_L^{-1}(1-q) \,|\, X \leq F_X^{-1}(p)),$$

where the probabilities p and q specify the severity of the factor stress and the confidence level applied to the portfolio loss distribution. In the following analysis, it will often be convenient to work with the survivorship distribution $P := -L$ instead of the loss distribution L, i.e. losses are represented by negative and not by positive numbers. In terms of the survivorship distribution, we can write[6]

$$FC_X(p,q) = \mathbb{P}(P \leq F_P^{-1}(q) \,|\, X \leq F_X^{-1}(p)). \tag{3}$$

The Factor Concentration $FC_X(p,q)$ specifies the probability that a loss is above the $(1-q)$-quantile of the loss distribution if a stress of the form $\{X \leq F_X^{-1}(p)\}$ is applied to the systematic factor X. The Factor Concentration divided by q, $FC_X(p,q)/q$, gives the relative change in the probability

$$\mathbb{P}(L \geq F_L^{-1}(1-q)) = \mathbb{P}(P \leq F_P^{-1}(q))$$

under the stress $\{X \leq F_X^{-1}(p)\}$. This notion of concentration risk has the advantage that it is completely independent of the specification of the credit risk model and the way that

[6]In order to ensure that equality (3) is also satisfied for non-continuous distributions L and P, the quantile $F_P^{-1}(q)$ of P has to be defined as the upper q-quantile $\inf\{x \in \mathbb{R} \mid \mathbb{P}(P \leq x) > q\}$ whereas the quantile $F_L^{-1}(1-q)$ of L is defined as the lower $(1-q)$-quantile $\inf\{x \in \mathbb{R} \mid \mathbb{P}(L \leq x) \geq 1-q\}$.

default dependencies are parameterized, e.g. whether asset or default correlations are used. In fact, it can be applied to factor models for market and operational risk as well. The parameters p and q provide the flexibility to analyze stress tests of different severity and to focus on specific parts of the distribution. Because the inequalities are expressed in quantile space, the Factor Concentration of X is largely independent of the marginal distributions of X and P, thus focusing on the dependence of the variables. In particular, if X and P are continuously distributed variables with copula C then

$$FC_X(p,q) = \frac{C(p,q)}{p}. \tag{4}$$

In a multi-factor credit portfolio model, FC_X usually is a rather complex object. Its analysis requires a simulation-based methodology for stress testing as presented in the previous section. In special cases, however, the calculation of FC_X is straightforward. Assume that X and P are independent. Then $FC_X(p,q)$ is independent of p and for large portfolios we have[7]

$$FC_X(p,q) \;=\; \mathbb{P}(P \leq F_P^{-1}(q)) \;\approx\; q. \tag{5}$$

The opposite extreme is perfect dependence of X and P, e.g. X and P are comonotonic or countermonotonic variables. Intuitively, this means that there exist monotonic functions f and g and a random variable V such that $X = f(V)$ and $P = g(V)$. If both functions are non-decreasing, X and P are comonotonic.[8] The most popular example of comonotonic variables in credit risk are the survivorship distribution P and the unique systematic factor X in the one-factor Vasicek model:

$$P \;=\; g(X) \;=\; -\,\Phi\left(\frac{\Phi^{-1}(pd) - \sqrt{\rho}X}{\sqrt{1-\rho}}\right)$$

and g is an increasing function applied to the systematic factor X. Hence, for probability q,

$$F_P^{-1}(q) = g(F_X^{-1}(q))$$

[7]In the general case, $\mathbb{P}(P \leq F_P^{-1}(q)) \geq q$. If P has a continuous distribution then $\mathbb{P}(P \leq F_P^{-1}(q)) = q$ and equality holds in (5).

[8]For a formal definition of these concepts, we refer to Embrechts, McNeil, and Straumann (2002).

and therefore (except in the degenerate case $\rho = 0$)

$$\begin{aligned} FC_X(p,q) &= \mathbb{P}(P \leq F_P^{-1}(q) \,|\, X \leq F_X^{-1}(p)) \\ &= \mathbb{P}(g(X) \leq g(F_X^{-1}(q)) \,|\, X \leq F_X^{-1}(p)) \\ &= \mathbb{P}(X \leq F_X^{-1}(q) \,|\, X \leq F_X^{-1}(p)). \end{aligned}$$

As a consequence, for continuous comonotonic X and P

$$\begin{aligned} &FC_X(p,q) = 1 && \text{if } p \leq q \\ &FC_X(p,q) = q/p && \text{if } p > q. \end{aligned}$$

In the same way we obtain for continuous countermonotonic X and P

$$\begin{aligned} &FC_X(p,q) = (p+q-1)/p && \text{if } p+q \geq 1 \\ &FC_X(p,q) = 0 && \text{if } p+q < 1. \end{aligned}$$

Figure 4 displays the Factor Concentration

$$FC_X(0.4,q), \qquad q \in [0,1]$$

for comonotonic, independent and countermonotonic factors in the stress scenario $X \leq F_X^{-1}(0.4)$. The curve represented by triangles is the Factor Concentration $FC_X(0.4,q)$ of the automotive factor X calculated in the case study in section 4.3. The Factor Concentrations of the comonotonic and countermonotonic variables provide the boundaries for values of $FC_X(p,q)$, the triangle above the diagonal is relevant for the identification of risk concentrations.

Another interpretation of Factor Concentration is obtained by reversing the roles of X and L: by Bayes rule[9]

$$\begin{aligned} FC_X(p,q) &= \mathbb{P}(L \geq F_L^{-1}(1-q) \,|\, X \leq F_X^{-1}(p)) \\ &= \frac{\mathbb{P}(L \geq F_L^{-1}(1-q) \wedge X \leq F_X^{-1}(p))}{\mathbb{P}(X \leq F_X^{-1}(p))} \cdot \frac{\mathbb{P}(L \geq F_L^{-1}(1-q))}{\mathbb{P}(L \geq F_L^{-1}(1-q))} \\ &\approx \mathbb{P}(X \leq F_X^{-1}(p) \,|\, L \geq F_L^{-1}(1-q)) \cdot \frac{q}{p}. \end{aligned} \tag{6}$$

It is worth noticing that in a Merton-type credit portfolio model of the form (1) there is a close formal relationship between (6) and risk contributions under the risk measure

[9]Equality in (6) holds for continuous X and L.

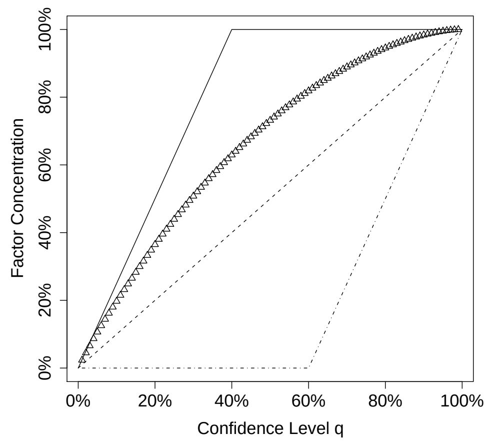

Figure 4: $FC_X(0.4, q)$ for comonotonic, independent and countermonotonic factors and for the automotive factor (triangles) in the case study.

Expected Shortfall: the Expected Shortfall contribution (w.r.t. confidence level $1-q$) of the i-th loan can be approximated by

$$\mathbb{E}(L_i \,|\, L \geq F_L^{-1}(1-q)) = \mathbb{P}(Y_i \leq \Phi^{-1}(p_i) \,|\, L \geq F_L^{-1}(1-q)) \cdot l_i.$$

In both concepts, the quantification of risk is based on conditional probabilities in the tail $\{L \geq F_L^{-1}(1-q)\}$. For allocating Expected Shortfall, the probability is calculated that the ability-to-pay variable Y_i is below the PD-threshold $\Phi^{-1}(p_i)$. The risk concentration in the systematic factor X is derived from the conditional probability of $\{X \leq F_X^{-1}(p)\}$.

5.2 Factor Concentration and Tail Dependence

The asymptotic behaviour of Factor Concentration is closely linked to Lower Tail Dependence, a well-known concept for quantifying the dependence of two random variables (see, for instance, McNeil, Frey and Embrechts (2005) or Malevergne and Sornette (2002)): the Lower Tail Dependence of two random variables X and Y is defined by

$$\lambda(X,Y) := \lim_{u \to 0^+} \mathbb{P}(Y \leq F_Y^{-1}(u) \,|\, X \leq F_X^{-1}(u)).$$

As a consequence of (4), Lower Tail Dependence is an asymptotic property of the copula C for continuously distributed variables X and Y. It can be rewritten as

$$\lambda(X,Y) = \lim_{u \to 0^+} \frac{C(u,u)}{u}$$

and its value is known explicitly for a large number of copulas. An extensive body of literature is available on this concept. Furthermore, graphical methods exist for detecting tail dependence (Chi-plots, K-plots) which could potentially be used for Factor Concentration as well.

It immediately follows from the definition of Factor Concentration that the limit

$$\lim_{u \to 0^+} FC_X(u,u)$$

equals the Lower Tail Dependence of the risk factor X and the survivorship distribution P, e.g.

$$\begin{aligned} \lambda(X,P) &= \lim_{u \to 0^+} \mathbb{P}((P \le F_P^{-1}(u) \,|\, X \le F_X^{-1}(u)) \\ &= \lim_{u \to 0^+} FC_X(u,u). \end{aligned}$$

While tail dependence is an asymptotic concept, we prefer to view FC as a function of its variables p and q for the purpose of managing the risk of a credit portfolio. In that way, the impact of various stresses on different parts of the loss distribution can be analyzed. In addition, we keep the freedom to set p and q independently. This is an important advantage because in typical applications the probability of a stress scenario will be far greater than the confidence level of the loss distribution we are interested in, e.g. $q = 0.02\%$ and $p = 5\%$.

5.3 Dynamic concentration risk

So far, we have focused on concentration risk which currently exists in a credit portfolio, and have defined the notion of Factor Concentration FC as an indicator. Rather than viewing concentration risk as a fixed or static property of the credit portfolio, we will now analyze how it varies in a stress scenario, i.e. we adopt a dynamic view of concentration risk. More precisely, under certain circumstances such as market stress events, even a formerly well-diversified portfolio can become concentrated due to the deterioration or default of certain parts of the portfolio. In the same way that Factor Concentration FC can be viewed as a sensitivity or first "derivative" of the loss distribution with respect to a particular systematic factor, the second derivative measures how risk concentrations change under stress.

The analysis of dynamic concentration risk can be embedded into our framework in a natural way:

1. Calculate the Factor Concentration for factor A in the original setup,
2. and repeat the calculation after stressing a different factor B.
3. The difference represents the change in Factor Concentration for factor A due to a stress in factor B.

In this application the stress applied to factor B would typically originate from an economic scenario as described in section 4, while its impact on the Factor Concentration of factor A is quantified through the function $FC_A(p, q)$.

Changes in concentration due to market stress events could be viewed as second order effects and therefore less important for risk management. However, it might be useful to be aware of those potential changes in concentration, in order to develop strategies in advance that make the portfolio more robust against stress events.

6 Conclusion

In this paper, we have presented a general framework for stressing risk concentration in credit portfolios. Starting from the idealized IRB portfolio model, we have discussed the concepts of name and sector concentration and have demonstrated that a multi-factor model is needed as the basis for stressing sector concentration.

The proposed approach to stressing sector concentration uses economic downturn scenarios or market shocks as a starting point. The scenarios are then implemented in a way that is consistent with the quantitative framework (e.g. without destroying the dependence structure of risk factors in the model). This is achieved by translating the economic stress scenarios into constraints on the systematic factors and on the state space of the model. The main prerequisite here is that the systematic factors of the credit portfolio model can be linked to economic variables.

Our stress testing methodology detects concentrations in distinct but highly correlated sectors, as demonstrated in a case study: while stressing a particular systematic factor has the largest impact on creditors in this sector, it still has a significant (though less pronounced) effect on creditors outside the sector.

Although the methodology has been developed in a particular factor model, the main concept - stressing sector concentration through a truncation of the distribution of the risk factors - is completely independent of the model specification and the way that default dependencies are parameterized, e.g. whether asset or default correlations are used. The mathematical formalization of the concept of Factor Concentration and the analysis of its basic properties form an important part of the paper.

We have calculated Expected Loss and VaR conditional on stress events. The latter is needed for assessing future economic capital requirements under stress. We have also outlined how the impact of a stress scenario on regulatory capital can be estimated by recalculating the Basel II formula with the stressed PDs from the multi-factor model.

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