A Fundamental Look at Economic Capital and Risk-Based Profitability Measures

Sebastian Fritz, Michael Kalkbrener, Wilfried Paus
Deutsche Bank AG, Credit Risk Management, Risk Analytics & Instruments*

July 29, 2004

1 Summary

Economic Capital and risk-based profitability measures like RAROC and Economic Profit are widely used throughout the finance industry. As their definitions have evolved over time, they - in our view - appear to lack a formal foundation and, at times, guidance for their application.

In this article, we take a step back to look at Economic Capital requirements and profitability measures from a slightly more rigid perspective - elaborating on procedures to determine justifiable capital requirements as well as linking their measures to modern portfolio theory.

The paper is structured as follows:

- Section 2 discusses if and how the right amount of Economic Capital for a bank can be determined.
- In Section 3, we present a formal description of the calculation of Economic Capital with some focus on its allocation.
- Section 4 deals with the integration of Economic Capital into popular risk-based profitability measures.
- In Section 5, we derive an alternative risk-based profitability measure based on the Capital Asset Pricing Model and study, how it ties in with profitability measures currently used.
- Finally, a summary of our main results concludes the article in Section 6.

2 Determining the Appropriate Level of Economic Capital

What are the main risks faced by a bank? What is the right amount of capital to cover those risks? How can capital be factored into an overall profitability measure that is applied to transactions, client relationships, business divisions and the bank as a whole?

These are the issues one comes across when implementing a risk-based profitability framework in a bank. Many banks use RAROC/RORAC or Economic Profit based measures (or a

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of Deutsche Bank AG.

combination of both). These measures are based on the assumption that risk is quantifiable and can therefore be used for determining profitability.

However, Capital Markets seem to tell the following story: the profitability figure often used for public disclosure is Net Return on Equity (RoE), since this concept is independent of in-house definitions of Economic Capital requirements. Price over book ratios for many large US-based or European banks can be explained to a large extent by the simple formula:

$$\frac{\text{Price}}{\text{Book}} = \frac{\text{Return on Equity}}{\text{Cost of Equity}}$$

when applying a Cost of Equity of $\approx 10\%$. Does that mean, that Capital Markets simply apply an average risk premium (e.g., 10% - LIBOR) when assessing the profitability of a bank?

Since banks are interested in stable or - even better - steadily increasing RoE numbers, management will usually support any legal method to stabilize earnings. We think that risk-adjusted profitability measures give the right incentives to generate long-term stable RoE numbers. In this article, we propose a methodology for determining Economic Capital requirements and for integrating Economic Capital into risk-adjusted profitability measures.

Any bank faces various uses (and potential misuses) of its Economic Capital:

1. preserving the bank's assets against very extreme events (stakeholder view);
2. preserving the bank's assets / deposits against systemic shock (regulatory view);
3. allocating the right amount of capital to a transaction in order to facilitate a risk/return analysis based on the risk appetite of the institution (shareholder view); and
4. giving an indication for line management of the riskiness / loss potential of certain transactions (management view).

It seems to be accepted in the industry that these different requirements can be covered by applying risk measures based on different tolerance levels to the loss distribution of the bank. In consequence, a bank could face more than one capital target and/or ratio to optimise:

1. the stakeholder, potentially backed by rating agencies, may prefer a very high confidence level - e.g. 99.99% for a AAA rating.
2. regulators seem to be satisfied with a confidence level of 99.90%[1].
3. shareholders may be interested in using a level of confidence in line with their risk appetite of the investment - say 99.80% for a BBB rated bank.
4. line management may rather want to see a level of confidence in the range of 90 to 95 % reflecting a 1 in 10 to 20 years event. Such a number could be seen as an average extreme loss an experienced manager may have come across during his professional career.

But what is a correct tolerance level for Economic Capital? According to Bessis (1998), the Economic Capital of a bank should reflect its target rating:

[1] According to the paper of the Bank of International Settlements - "International Convergence of Capital Measurement and Capital Standards" so-called Basel 2 Risk Weight Formula - http://www.bis.org/publ/bcbs107.pdf

There is a direct link between capital, risk, tolerance level and the bank's default risk. The choice of the level of Economic Capital has to be consistent with the strategy of the bank. The most relevant rule is to choose the capital that provides the desired rating given the risks. Since ratings can be associated to average default rates within each class, the tolerance level should be identical to that associated with the targeted rating.

However, as Matten (1996) explains, "overcapitalisation" driven by a very conservative target rating has potential downsides:

Against these (the capital requirements) must be set the target return which management wishes to achieve. This will be driven by the market's expectations of returns - exceeding the market's expectations will result in an increase in shareholder value, whereas failing to meet those expectations will result in a destruction of value. The higher the amount of capital which a bank maintains, the higher the profit it will have to earn in order to make the target return.

These conflicting objectives are difficult to balance. We propose the following boundary conditions on Economic Capital requirements:

1. the bank-internal tolerance level should be above minimum regulatory requirements, say the 99.90% confidence level.

2. the tolerance level should be above the confidence level associated with the current rating of the bank.[2]

3. if the first two conditions are met, risk tolerance should be at a level which utilises more than 90 % of available equity (keeping the rest as a buffer) in order to achieve maximum utilisation of shareholder's money.

Once an adequate tolerance level is defined, should it be held constant or varied over an economic cycle? We favour to hold the tolerance level constant since a change in tolerance level will not only change the overall Economic Capital requirement, it may also impact capital allocation and therefore the relative profitability of different assets. A lowering of the tolerance level should only be considered as last resort to prevent under-capitalisation. In any disclosure, it would certainly be understood as a material increase in risk appetite of the bank.

If at a given tolerance level the utilisation of shareholder's equity drops, a bank is faced with three alternatives: it can

1. invest the released resource into new business (organic growth, acquisitions) in order to increase the revenue base.

2. give some of the capital back to the shareholders (share buy-back).

3. increase the tolerance level in order to achieve a rating upgrade. This would certainly be interpreted as a reduction in risk appetite of the bank.

In consequence, while regular changes of the tolerance level could be viewed as a means to smooth RoE numbers, we suggest keeping the tolerance level stable as long as the bank does not fundamentally change its strategy.

[2] There may be an issue for banks whose current rating ranks below the regulatory threshold.

3 Calculating and Allocating Economic Capital

In the previous section we have discussed Economic Capital requirements and tolerance levels of banks. The objective of this section is to identify specific risk measures for the calculation and allocation of Economic Capital. The analysis will be done in a portfolio context, i.e., we will determine Economic Capital for a portfolio $\mathcal{P}$ and allocate Economic Capital to its subportfolios $\mathcal{P}_1, \dots, \mathcal{P}_n$.[3] In this section, portfolios will be represented by their loss variables. More precisely, the loss variable $X_{\mathcal{P}}$ of portfolio $\mathcal{P}$ is a real-valued function on the probability space $(\Omega, \mathbb{P})$ of all scenarios and $X_{\mathcal{P}}(\omega)$ specifies the loss of portfolio $\mathcal{P}$ in scenario $\omega \in \Omega$.

3.1 Economic Capital Calculation

Economic Capital is a measure designed to state with a high degree of certainty the amount of equity capital needed to absorb unexpected losses. The exact meaning of "high degree of certainty" depends on the risk tolerance of the bank. A popular rule is to choose the tolerance level that reflects the average default rate associated with the rating of the bank. This rule immediately translates into a definition of Economic Capital based on Value-at-Risk[4]: for example, if the current rating of a bank is AA+ associated with a default rate of 0.02% then the Economic Capital is determined by the Value-at-Risk of the bank's loss distribution at level $\alpha = 99.98\%$.

The simple link between ratings and Value-at-Risk based Economic Capital is one reason why this specific Economic Capital methodology has become so popular and has even achieved the high status of being written into industry regulations. The standard procedure for most risk types is to specify the Economic Capital of $\mathcal{P}$ as Value-at-Risk minus Expected Loss:

$$\mathrm{EC}(\mathcal{P}) := \mathrm{VaR}_\alpha(X_{\mathcal{P}}) - \mathrm{E}(X_{\mathcal{P}}), \tag{1}$$

where $\mathrm{E}(X_{\mathcal{P}})$ denotes the mean of the loss variable $X_{\mathcal{P}}$ of the portfolio $\mathcal{P}$.

It is well-known, however, that Economic Capital based on Value-at-Risk has serious disadvantages. As long as the loss variables of portfolios are normally distributed, the Value-at-Risk methodology encourages diversification, i.e.

$$\mathrm{VaR}(X + Y) \leq \mathrm{VaR}(X) + \mathrm{VaR}(Y) \tag{2}$$

for normally distributed loss variables X and Y. For most portfolios the normal distribution assumption is not justified and in this case subadditivity (2) no longer holds for Value-at-Risk. Diversification, which is commonly considered as a way to reduce risk, may increase Value-at-Risk and therefore Economic Capital.

It has therefore been proposed in a number of papers to replace Value-at-Risk in (1) by more appropriate risk measures ρ:

$$\mathrm{EC}(\mathcal{P}) := \rho(X_{\mathcal{P}}) - \mathrm{E}(X_{\mathcal{P}}). \tag{3}$$

[3] $\mathcal{P}$ can also be interpreted as a bank with business units $\mathcal{P}_1, \dots, \mathcal{P}_n$.

[4] In this article, the Value-at-Risk $\mathrm{VaR}_\alpha(X)$ at confidence level $\alpha \in (0,1)$ is defined as the smallest α-quantile of X, i.e.,

$$\mathrm{VaR}_\alpha(X) := \inf\{x \in \mathbb{R} \mid \mathbb{P}(X \leq x) \geq \alpha\}.$$

But what are the properties of good risk measures? To answer this question, Artzner et al. (1997, 1999) presented the following set of axioms.

Coherency axioms for risk measures
Let V denote the linear space of all portfolio loss variables and let ρ be a function from V to $\mathbb{R}$, i.e., $\rho(X)$ denotes the risk capital of a portfolio $X \in V$. The risk measure ρ is called coherent if it is

monotonic:	$X \leq Y \;\Rightarrow\; \rho(X) \leq \rho(Y)$	$\forall X, Y \in V$,
translation invariant:	$\rho(X + a) \;=\; \rho(X) + a$	$\forall a \in \mathbb{R},\; X \in V$,
positively homogeneous:	$\rho(aX) \;=\; a \cdot \rho(X)$	$\forall a \geq 0,\; X \in V$,
sub-additive:	$\rho(X + Y) \;\leq\; \rho(X) + \rho(Y)$	$\forall X, Y \in V$.

The most popular class of coherent risk measures is Expected Shortfall (see, for instance, Rockafellar and Uryasev, 2000; Acerbi and Tasche, 2002). For practical purposes, the Expected Shortfall of X at level $\alpha \in (0, 1)$, denoted by $\mathrm{ES}_\alpha(X)$, can be defined by

$$\mathrm{E}(X | X > \mathrm{VaR}_\alpha(X)) = (1 - \alpha)^{-1} \mathrm{E}(X \cdot \mathbf{1}_{\{X > \mathrm{VaR}_\alpha(X)\}}). \tag{4}$$

Intuitively, Expected Shortfall can therefore be interpreted as the average of all losses above a given quantile of the loss distribution. Its exact definition[5] takes care of jumps of the loss distribution at its quantile. More general techniques for specifying coherent risk measures are given in Artzner et al. (1999), Delbaen (2000, 2002) and Acerbi (2004).

3.2 Capital Allocation

The coherency axioms provide an excellent framework for the theoretical analysis of risk measures. We now turn to the second problem: the allocation of Economic Capital either to subportfolios or to business units. More formally, assume that a risk measure ρ has been fixed and that the Economic Capital of $\mathcal{P}$ has been calculated according to (3). The objective is to distribute $\mathrm{EC}(\mathcal{P})$ to its subportfolios $\mathcal{P}_1, \ldots, \mathcal{P}_n$.

Again, we propose an axiomatic framework for the analysis and comparison of allocation schemes.[6] A capital allocation is a function Λ from $V \times V$ to $\mathbb{R}$. Its interpretation is, that $\Lambda(X, Y)$ represents the capital allocated to the portfolio X - considered as a subportfolio of portfolio Y. Λ is called a capital allocation with respect to a risk measure ρ if it universally satisfies $\Lambda(X, X) = \rho(X)$, i.e., the capital allocated to X (considered as stand-alone portfolio) is the risk capital $\rho(X)$ of X. We propose the following requirements for a capital allocation scheme in a bank.

3.2.1 Allocation Axioms

Let Λ be a capital allocation with respect to ρ, i.e.,

$$\forall X \in V \quad \Lambda(X, X) = \rho(X).$$

[5]More precisely: $\mathrm{ES}_\alpha(X) := (1 - \alpha)^{-1}(\mathrm{E}(X\, \mathbf{1}_{\{X > \mathrm{VaR}_\alpha(X)\}}) + \mathrm{VaR}_\alpha(X) \cdot (\mathbb{P}(X \leq \mathrm{VaR}_\alpha(X)) - \alpha))$. It is easy to see that for most loss distributions the Expected Shortfall ES_α is dominated by the first term given by (4). The second term is added to ensure coherence.

[6]Theoretical and practical aspects of different allocation schemes have been analyzed in a number of papers, for instance in Hallerbach (2003), Schmock and Straumann (1999), Delbaen (2000), Denault (2001), Tasche (1999, 2002). The allocation methodology presented in this article has been developed in Kalkbrener (2002).

1. **Linearity.** The capital allocated to a union of subportfolios is equal to the sum of the capital amounts allocated to the individual subportfolios. In particular, the risk capital of a portfolio equals the sum of the risk capital of its subportfolios. More formally, Λ is called linear if

$$\forall a, b \in \mathbb{R}, X, Y, Z \in V \quad \Lambda(aX + bY, Z) = a\Lambda(X, Z) + b\Lambda(Y, Z).$$

2. **Diversification.** The capital allocated to a subportfolio X of a larger portfolio Y never exceeds the risk capital of X considered as a stand-alone portfolio: Λ is called diversifying if

$$\forall X, Y \in V \quad \Lambda(X, Y) \leq \Lambda(X, X).$$

3. **Continuity.** A small increase in a position does only have a small effect on the risk capital allocated to that position: Λ is called continuous at $Y \in V$ if

$$\forall X \in V \quad \lim_{\epsilon \to 0} \Lambda(X, Y + \epsilon X) = \Lambda(X, Y).$$

It is interesting that a linear and diversifying capital allocation, which is continuous at a portfolio $Y \in V$, is uniquely determined by its associated risk measure, i.e., the diagonal values of Λ. More specifically, given the portfolio Y, the capital allocated to a subportfolio X of Y is the derivative of the associated risk measure ρ at Y in the direction of X:

$$\forall X \in V \quad \Lambda(X, Y) = \lim_{\epsilon \to 0} \frac{\rho(Y + \epsilon X) - \rho(Y)}{\epsilon}. \tag{5}$$

This result shows that the three axioms are sufficient to uniquely determine a capital allocation scheme. It remains to be shown that capital allocations that satisfy the axioms do exist. It turns out that the existence of linear, diversifying capital allocations is mathematically equivalent to certain properties of the associated risk measure: there exists a linear and diversifying capital allocation with respect to ρ if and only if ρ is sub-additive and positively homogeneous.

So far, we have focused on allocations Λ with respect to arbitrary risk measures ρ. Using (1), Λ can easily be transformed into an allocation scheme for Economic Capital: the Economic Capital allocated to subportfolio $\mathcal{P}_i$ equals

$$\mathrm{EC}(\mathcal{P}_i, \mathcal{P}) := \Lambda(X_{\mathcal{P}_i}, X_{\mathcal{P}}) - \mathrm{E}(X_{\mathcal{P}_i}),$$

where $X_{\mathcal{P}_i}$ and $X_{\mathcal{P}}$ are the loss variables of subportfolio $\mathcal{P}_i$ and portfolio $\mathcal{P}$.

3.2.2 Examples

The risk measure Expected Shortfall (ES for short) is coherent and therefore sub-additive and positively homogeneous. Application of the above theorems yields a linear, diversifying allocation scheme with associated risk measure ES: the Expected Shortfall contribution of subportfolio X of Y defined as[7]

$$\mathrm{E}(X | Y > \mathrm{VaR}_\alpha(Y)) = (1 - \alpha)^{-1} \mathrm{E}(X \cdot \mathbf{1}_{\{Y > \mathrm{VaR}_\alpha(Y)\}}). \tag{6}$$

[7] Precisely: $\mathrm{ESC}_\alpha(X, Y) := (\mathrm{E}(X \cdot \mathbf{1}_{\{Y > \mathrm{VaR}_\alpha(Y)\}}) + \beta_Y \cdot \mathrm{E}(X \cdot \mathbf{1}_{\{Y = \mathrm{VaR}_\alpha(Y)\}}))/(1 - \alpha)$ with $\beta_Y := \frac{\mathbb{P}(Y \leq \mathrm{VaR}_\alpha(Y)) - \alpha}{\mathbb{P}(Y = \mathrm{VaR}_\alpha(Y))}$ which again is dominated by (6).

Hence, the Expected Shortfall contribution of a subportfolio can be considered as its average contribution to portfolio losses exceeding the quantile $\mathrm{VaR}_\alpha(Y)$.

Value-at-Risk, on the other hand, is not a sub-additive risk measure. As a consequence, there do not exist linear diversifying capital allocations for VaR in general. Also, from a practical point of view, the allocation of Value-at-Risk is a difficult problem. For market risk applications, Hallerbach (1999) suggests allocation based on the directional derivative (5) with $\rho = \mathrm{VaR}_\alpha$. This technique works well in a sufficiently continuous setting (see Tasche (1999) and Gouriéroux et al. (2000) for criteria which ensure existence of (5)). However, in non-continuous or even discrete models directional derivatives usually do not exist or they are not continuous and highly unstable in α.

The standard solution in credit portfolio modeling is to allocate portfolio VaR proportionally to the loss covariances

$$\mathrm{Cov}(X_1, X), \ldots, \mathrm{Cov}(X_n, X).$$

This allocation technique, called volatility allocation, is the natural choice in classical portfolio theory where portfolio risk is measured by standard deviation (or volatility).

In general, combining volatility allocation with Value-at-Risk works well as long as all loss distributions are close to normal. For credit portfolios, however, it does not: the capital allocated to a subportfolio X of Y may be greater than the risk capital of X considered as a stand-alone portfolio, the capital charge of a loan may even be higher than its exposure (Kalkbrener et al., 2004).

4 Common Profitability Measures

The purpose of the previous section was to develop concepts for calculating and allocating Economic Capital. The objective of this section is to use Economic Capital for valuing transactions in a portfolio and assessing the shareholder value created by business units.

Profitability assessment by banks is commonly carried out by expressing the risk-return relationship as simple rational functions of risk and return components. The two basic variants of such so-called risk-adjusted ratios are known as RORAC or RAROC, respectively. Both are refinements of the classic Return on Equity (ROE) ratio.

4.1 Reference Capital

Before going into more detail, we pick up on the issues raised in Section 2 regarding the role of capital in a bank.

While the Equity Capital is an important and undisputable capital quantity, it does not tell anything about the riskiness of the businesses it is used to support. When measuring profitability from a risk-return perspective, it is therefore important to relate returns to a risk-driven capital measure instead. The natural choice for this is Economic Capital. Another option is to use regulatory capital, which, in particular after the refinements under the Basel II accord, is also risk-sensitive. In view of its limitations (e.g., not reflecting diversification effects), however, we believe that it should enter any profitability measure rather as a cost than a risk (capital) measure.

Naturally, any capital measure needs to be related to the shareholder's equity capital. A common procedure to achieve this is as follows, taken from Deutsche Bank's Annual Report 2002:

We rely on our book capital to absorb any losses that result from the risks we assume in our businesses. Book capital is defined as the amount of equity capital that appears in our balance sheet. We use economic capital as our primary tool to allocate our book capital among our businesses. We also use it to assess their profitability and their relative abilities to employ capital efficiently.

4.2 RAROC Ratios

The RORAC measure, or Return On Risk-Adjusted Capital, is a profitability calculated on risk-based capital. In its strict definition RAROC, or Risk-Adjusted Return On Capital, adjusts the return for risk (for instance by calculating margins net of statistical defaults) and relates the result to the equity capital held.

In order to establish a comprehensive risk-return relationship, however, both these adjustments are necessary. Any Expected Loss should be deducted from anticipated margin income, and the resulting risk-adjusted return should be related to the capital necessary to absorb potential unexpected losses. Consequently, one should merge the two ideas behind RORAC and RAROC into one ratio. Following a general trend throughout the financial industry, we continue to refer to it as RAROC exclusively from now on:

$$\text{RAROC} := \frac{\text{Risk-Adjusted Return}}{\text{Economic Capital}}.$$

Herein, the numerator Risk-Adjusted Return comprises the following components:

- Gross Revenues[8] of a transaction / customer relationship,
- Administration Costs (determined by the cost structure and the workflow within the bank),
- Expected Loss (calculated consistently with the Economic Capital component given in the RAROC denominator).

It is obvious from the above that, subject to the existence of a suitable allocation of Economic Capital, RAROC may be calculated on the portfolio as well as on the transaction level.

Even though the general definition of RAROC given in the above is commonly agreed within the industry, its detailed calculation has not been fixed yet. As work flows, data availability and in particular the business to be evaluated may vary dramatically across institutions, no uniform definitions for these quantities, which may be applicable to all banks alike, exist. Naturally, this bears the risk of mushrooming of different methodological approaches used across banks or even within one institution.

4.3 Economic Profit

RAROC is a dimensionless risk-return measure. It ranks transactions, customers or business lines by a performance rate (on allocated Economic Capital) rather than their performance in monetary terms. But, fixing a minimum level of RAROC required by the bank, the so-called

[8]Net of refinancing costs.

hurdle rate h, automatically gives rise to a profitability measure in currency units. We define the Economic Profit[9] EP by

$$\text{EP} = \text{Risk-Adjusted Return} - h \cdot \text{Economic Capital}. \tag{7}$$

The product of the hurdle rate and the Economic Capital is commonly referred to as the Cost of Capital, as this quantity may be regarded as the yearly interest one would have to pay for borrowing Economic Capital at h. The benchmark for h is the price of risk in the capital market from the shareholder perspective. When employing the equity capital allocation procedure described in Section 4.1, it can be derived directly from the ROE expectation of the shareholders and the ratio of the bank's Economic Capital and its equity capital:

$$h \approx \text{ROE-target} \cdot \frac{\text{Equity Capital}}{\text{Economic Capital}}.$$

The Economic Profit thus quantifies the monetary amount in excess (below) the Cost of Capital generated by a transaction (or a set thereof).

Two transactions may have equal RAROCs while the riskier one generates a higher income or loss, given by their respective EPs. Hence, regarding performance objective setting, EP is the superior measure to RAROC as it quantifies true profits and losses. Therefore, in order to improve an overall profitability rate, it is always possible to eliminate those transactions that are less profitable and to keep only the others. In such cases, the profitability (as a percentage) increases at the expense of size. Usually, both size and profitability should be managed. Using a target value of Economic Profit makes explicit the feasibility of such trade-offs and puts some emphasis on the volume of operations.

5 Alternatives

While the RAROC / EP concept appears intuitive and is easily communicated due to its cash-flow oriented view (all components can be interpreted as individual cash flows), it is not clear whether it covers all aspects of interdependence of these cash-flows. Moreover, it does not tell the user what an optimal, yet realistic return of a given portfolio could have been.

In this section, we will therefore focus rather on benchmarking a given portfolio to an optimal portfolio from a risk-return perspective. Hereby, we use the Capital Asset Pricing Model (Sharpe, 1964; Lintner, 1965) to derive an alternative profitability measure in Section 5.3 and subsequently compare it with the Economic Profit introduced in Section 4.3.

5.1 Problems with Economic Profit

It is one of the fundamental tenets of modern portfolio theory (the underpinning of all modern financial mathematics) that there is a relationship between the market's expected or required return and the uncertainty surrounding that return. Profitability measures should incorporate this relationship directly alongside the quantification of the loss potential due to defaults or other risk events.

RAROC and Economic Profit, being simple financial ratios or a linear combination of the risk-adjusted return and the (allocated) Economic Capital, can not fully recognize the volatility

[9] Also widely known as "Value Creation" or "(Shareholder) Value Added".

of returns. This is mainly due to the fact that the purpose of Economic Capital is to quantify extreme loss events: Economic Capital is inevitably focused on the tail and does not adequately reflect the volatility of returns across all scenarios.

A common workaround to still reflect the volatility of earnings is the following: The Economic Capital term comprises a term build to assess the so-called business risk. It is designed to capitalize unexpected losses caused by the break-away of revenue streams in specific business lines. Business Risk EC is frequently merged with the Economic Capital for Operational Risk, but its calculation, since driven by modelling P&L volatility, differs substantially from the approaches used to model Operational Risk.

In our view it is therefore advisable to, rather than deduct another cost component from the Economic Profit, one should take an integrated view of returns and risk.

5.2 Capital Asset Pricing Model

A widely accepted approach to modelling the dependency between market return and its uncertainty is given by the Capital Asset Pricing Model (CAPM), where uncertainty is expressed in the form of the standard deviation of the return. More precisely, the model states that the required return R_i on an investment can be determined by reference to the historical relationship between the investment and the market:

$$\mathrm{E}(R_i) = R_F + \beta_i(\mathrm{E}(R_M) - R_F),$$

where R_F refers to the risk-free rate, and R_M to the market return[10]. The expression $\mathrm{E}(R_M) - R_F$ refers simply to the excess return of the market over the risk-free rate (the risk premium). The beta is a factor for each individual stock which measures how closely the stock follows the market. It is defined as the covariance of R_i and R_M divided by the variance of R_M.

5.3 A CAPM Implied Profitability Measure

Our objective is to integrate the concept of Economic Capital into the Capital Asset Pricing Model and to derive a profitability measure in the extended model.

Let R be the return variable of a portfolio $\mathcal{P}$ which consists of n subportfolios $\mathcal{P}_1, \ldots, \mathcal{P}_n$ with return variables $R_1, \ldots, R_n$, i.e., $R = R_1 + \ldots + R_n$. We assume that Economic Capital has been calculated and allocated to each of the portfolios using the techniques presented in Section 3: the Economic Capital of the portfolio $\mathcal{P}$ is denoted by $\mathrm{EC}(\mathcal{P})$ and the capital allocated[11] to $\mathcal{P}_i$ is denoted by $\mathrm{EC}(\mathcal{P}_i, \mathcal{P})$.

As a prerequisite to applying the CAPM, we need to normalise the subportfolios under consideration to unit size. For this, we scale each subportfolio by the factor $\mathrm{EC}(\mathcal{P})/\mathrm{EC}(\mathcal{P}_i, \mathcal{P})$, such that all subportfolios have the same Economic Capital. The return of the i-th scaled subportfolio $\bar{\mathcal{P}}_i$ is then given by

$$\bar{R}_i := \frac{\mathrm{EC}(\mathcal{P})}{\mathrm{EC}(\mathcal{P}_i, \mathcal{P})} \cdot R_i.$$

[10]More precisely, R_F (R_M) is the return on an investment of one unit of cash in the risk-free asset (market portfolio).

[11]It is assumed that the capital allocation satisfies the linearity axiom in Section 3.2.1.

Note that the contributory Economic Capital of $\bar{\mathcal{P}}_i$ equals $\mathrm{EC}(\mathcal{P}) = (\mathrm{EC}(\mathcal{P})/\mathrm{EC}(\mathcal{P}_i, \mathcal{P})) \cdot \mathrm{EC}(\mathcal{P}_i, \mathcal{P})$. Furthermore,

$$R = \frac{\mathrm{EC}(\mathcal{P}_1, \mathcal{P})}{\mathrm{EC}(\mathcal{P})} \cdot \bar{R}_1 + \ldots + \frac{\mathrm{EC}(\mathcal{P}_n, \mathcal{P})}{\mathrm{EC}(\mathcal{P})} \cdot \bar{R}_n \tag{8}$$

$$\text{and} \quad 1 = \frac{\mathrm{EC}(\mathcal{P}_1, \mathcal{P})}{\mathrm{EC}(\mathcal{P})} + \ldots + \frac{\mathrm{EC}(\mathcal{P}_n, \mathcal{P})}{\mathrm{EC}(\mathcal{P})}. \tag{9}$$

The identities (8) and (9) can be interpreted in the following way: $\mathcal{P}$ is a portfolio consisting of investments $\bar{\mathcal{P}}_1, \ldots, \bar{\mathcal{P}}_n$. The total amount of capital invested in $\mathcal{P}$ is $\mathrm{EC}(\mathcal{P})$, the fraction invested in $\bar{\mathcal{P}}_i$ equals $\mathrm{EC}(\mathcal{P}_i, \mathcal{P})/\mathrm{EC}(\mathcal{P})$. Following the Capital Asset Pricing Model, the expected return of $\bar{\mathcal{P}}_i$ is now defined as

$$\left(R_F + (E_R - R_F) \cdot \frac{Cov(R, \bar{R}_i)}{Var(R)} \right) \cdot \mathrm{EC}(\mathcal{P}),$$

where E_R denotes the normalized mean of the portfolio return R, i.e. $E_R = \mathrm{E}(R)/\mathrm{EC}(\mathcal{P})$. By multiplying with $\mathrm{EC}(\mathcal{P}_i, \mathcal{P})/\mathrm{EC}(\mathcal{P})$ we obtain the expected return of the (unscaled) portfolio $\mathcal{P}_i$:

$$\left(R_F + (E_R - R_F) \cdot \frac{Cov(R, \bar{R}_i)}{Var(R)} \right) \cdot \mathrm{EC}(\mathcal{P}) \cdot \frac{\mathrm{EC}(\mathcal{P}_i, \mathcal{P})}{\mathrm{EC}(\mathcal{P})} =$$

$$R_F \cdot \mathrm{EC}(\mathcal{P}_i, \mathcal{P}) + (E_R - R_F) \cdot \mathrm{EC}(\mathcal{P}) \cdot \frac{Cov(R, R_i)}{Var(R)}.$$

Our profitability measure is derived from the formula above. More precisely, we propose to replace the expected portfolio return E_R by a target return $\tilde{h}$ set by the bank. Subtraction from $\mathrm{E}(R_i)$ yields the profitability measure

$$P(\mathcal{P}_i, \mathcal{P}) := \mathrm{E}(R_i) - \left(R_F \cdot \mathrm{EC}(\mathcal{P}_i, \mathcal{P}) + (\tilde{h} - R_F) \cdot \mathrm{EC}(\mathcal{P}) \cdot \frac{Cov(R, R_i)}{Var(R)} \right), \tag{10}$$

with the properties

$$P(\mathcal{P}_i, \mathcal{P}) \left\{ \begin{array}{c} = 0 \\ > 0 \\ < 0 \end{array} \right\} \text{if } \mathcal{P}_i\text{'s return } R_i \left\{ \begin{array}{c} \text{meets} \\ \text{exceeds} \\ \text{falls short of} \end{array} \right\} \text{its expectation.}$$

Observe that this profitability measure comprises two different allocation methods for Economic Capital, namely $\mathrm{EC}(\mathcal{P}_i, \mathcal{P})$ (an allocation complying with the axioms listed in Section 3.2.1, e.g., Expected Shortfall) and the volatility allocation $\mathrm{EC}(\mathcal{P}) \cdot \frac{Cov(R, R_i)}{Var(R)}$. In this sense, the CAPM implied profitability measure yields a synthesis of two capital allocation methods commonly thought to be suitable only for different perspectives (Hall, 2002).

5.4 Comparison with Economic Profit

We will now investigate the relationship between the CAPM implied profitability measure and Economic Profit. Recall from (7), that Economic Profit is defined by

$$\mathrm{EP} = \text{Risk-Adjusted Return} - h \cdot \text{Economic Capital},$$

where the Risk-Adjusted Return comprises the components Gross Revenues, Administration Costs and Expected Loss. Assuming that these components are covered by the return variable R_i, Economic Profit can be written as

$$\mathrm{EP}(\mathcal{P}_i, \mathcal{P}) = E(R_i) - h \cdot \mathrm{EC}(\mathcal{P}_i, \mathcal{P}) \tag{11}$$

Comparing terms on the right hand sides of (10) and (11) and choosing $\tilde{h} \equiv h$ (corresponding to setting the return hurdle to covering net capital costs[12]), exposes that the difference between $\mathrm{EP}(\mathcal{P}_i, \mathcal{P})$ and $P(\mathcal{P}_i, \mathcal{P})$ boils down to

1. the allocation mechanism applied to the capital cost term[13] and
2. an additional risk cost term $R_F \cdot \left(\mathrm{EC}(\mathcal{P}_i, \mathcal{P}) - \mathrm{EC}(\mathcal{P}) \cdot \frac{Cov(R,R_i)}{Var(R)}\right)$ in (10).

5.5 Analysis

We now analyse two extreme scenarios in order to illustrate the differences between Economic Profit and the CAPM implied profitability measure.

1. **"Break-even"**: $h = R_F \quad \Rightarrow \quad \mathrm{EP} \equiv P$.
2. **"High ambition"**: $h \gg R_F \searrow 0 \quad \Rightarrow \quad \mathrm{EP} \approx E(R) - h \cdot \mathrm{EC}(\cdot, \mathcal{P}),$
   $P \approx E(R) - h \cdot \mathrm{EC}(\mathcal{P}) \cdot \frac{Cov(R,\cdot)}{Var(R)}$.

The "Break-even" scenario is likely to hold for banks, which have set a low hurdle rate (state-owned institutions, focus on retail). In this scenario, Economic Profit and the CAPM implied profitability measure are identical.

The "High ambition" scenario occurs if either the risk-free rates are very low (ie low interest rates) or the hurdle set by the individual bank is very high. This is usually the case for banks with strong M&A and trading divisions. A high hurdle implies that allocated Economic Capital becomes less important for profitability, and that more emphasis is put on the contribution of transactions (subportfolios, business units) to the overall earnings volatility of the bank. In the "High ambition" scenario, the difference between Economic Profit and the CAPM implied profitability measure reduces to the allocation of Economic Capital. The latter implies that the banks in question optimise their risk-return best when allocating Economic Capital via a volatility allocation rather than an allocation associated with a coherent Economic Capital measure.

Under current market conditions (Summer 2004), risk-free rates around EURIBOR $\approx 2.5\%$ and hurdle rates after tax between 10% and 15% are common for large European banks. That ratio is approximately in line with an allocation of four units of volatility versus one unit of Expected Shortfall. In this case, Economic Profit based on volatility allocation is a reasonable approximation for optimising and stabilising risk-return.

In conclusion, the bigger the difference between the risk-free rate and the hurdle rate set by the bank the more weight is shifted from Expected Shortfall to volatility allocation when optimising risk-return.

[12]Note that the CAPM is based on the assumption of assets being available at no charge whereas the RAROC / EP framework does not.

[13]One must note, however, that this comparison assumes that the risk measure $\mathrm{EC}(\mathcal{P})$ is covering all aspects of risks due to the volatility of earnings. As discussed in Section 5.1, this is generally not the case for many Economic Profit measures used in the industry.

6 Conclusion

In this closing section, we summarise the key messages of this article.

Tolerance Level for Economic Capital
In order to ensure a sufficient protection from extreme risk events alongside avoiding overcapitalisation at the expense of the bank's shareholders, we suggest deriving a bank's individual Economic Capital requirement as follows:

- The bank-internal tolerance level should lie above both, the minimum regulatory requirement and the confidence level associated with the current rating of the bank.
- If this minimum condition is met, risk tolerance should be at a level, which utilises more than 90 % of available equity in order to achieve maximum utilisation of shareholder's money.

Once an adequate tolerance level is defined, it should be held constant - with appropriate action taken should the utilisation of shareholder's equity drop.

Coherent Risk Measures and Allocation
Calculating Economic Capital based on Value-at-Risk has, while being popular with banks and regulators alike, serious disadvantages - the major one being that it does not necessarily recognise diversification. Consequently, one should rather base the calculation of Economic Capital on a risk measure with properties matching intuition. The properties are given by the so-called coherency axioms, the most prominent class of risk measure satisfying these axioms is the one of Expected Shortfall measures.

A similarly fundamental approach can be taken in order to allocate Economic Capital to subportfolios or to business units compliant with intuition. Requiring basic properties of linearity, continuity and diversification uniquely determines an allocation to any given coherent risk measure.

Refined Profitability Measures
Risk-adjusted profitability measures based on Economic Capital give the right incentives to generate long-term stable RoE numbers. The common quantities RAROC and Economic Profit, being simple financial ratios or a linear combination of the risk-adjusted return and the (allocated) Economic Capital, however, can not fully recognise the volatility of returns.

In an attempt to take an integrated view of returns and risk, we applied the Capital Asset Pricing Model (CAPM) to derive an alternative profitability measure. Analysis shows that this CAPM implied profitability measure coincides with the classical Economic Profit if the bank's return on capital requirement is as low as the risk-free rate. As the bank's return target increases, however, the alternative profitability measure puts more emphasis on the volatility of earnings - a behaviour in line with both, the shareholder and the stakeholder perspective.

References

[1] C. Acerbi. Coherent representations of subjective risk-aversion. In J. Szegö (ed.): *Risk Measures for the 21st Century*, Wiley, 2004.

[2] C. Acerbi and D. Tasche. On the coherence of expected shortfall. *Journal of Banking and Finance* 26, 1487-1503, 2002.

[3] P. Artzner, F. Delbaen, J.-M. Eber and D. Heath. Thinking coherently. *RISK* Nov. 1997, 68-71.

[4] P. Artzner, F. Delbaen, J.-M. Eber and D. Heath. Coherent measures of risk. *Mathematical Finance* 9, 203-228, 1999.

[5] J. Bessis. *Risk Management in Banking.* Wiley, 1998.

[6] F. Delbaen. Coherent risk measures. Lecture notes, Scuola Normale Superiore di Pisa, 2000.

[7] F. Delbaen. Coherent risk measures on general probability spaces. In Sandmann, K. and Schönbucher, P.J. (eds): Advances in Finance and Stochastics - Essays in Honour of Dieter Sondermann, Springer, 2002.

[8] M. Denault. Coherent allocation of risk capital. *Journal of Risk* 4(1), 2001.

[9] Deutsche Bank AG. Annual Report, 2002.

[10] C. Gouriéroux, J.P. Laurent and O. Scaillet. Sensitivity analysis of value at risk. *Journal of Empirical Finance* 7, 225-245, 2000.

[11] C. Hall. Economic capital: towards an integrated risk framework. *RISK* (Oct. 2002), 33-38.

[12] W. Hallerbach. Decomposing portfolio value-at-risk: a general analysis. *Journal of Risk* 5(2), 1-18, 2003.

[13] M. Kalkbrener. An axiomatic approach to capital allocation. Technical Document, Deutsche Bank AG, Frankfurt, 2002.

[14] M. Kalkbrener, H. Lotter and L. Overbeck. Sensible and efficient capital allocation for credit portfolios. *RISK* (Jan. 2004), S19-S24.

[15] J. Lintner. Security prices, risk and maximal gains from diversification. *Journal of Finance* (Dec. 1965), 587-615.

[16] C. Matten. *Managing Bank Capital: Capital Allocation and Performance Measurement.* Wiley, 1996.

[17] R.T. Rockafellar and S. Uryasev. Optimization of conditional value-at-risk. *Journal of Risk* 2, 21-41, 2000.

[18] U. Schmock, and D. Straumann. Allocation of risk capital and performance measurement. Seminar presentations at Quantitative Methods in Finance 1999 Conference, University of Technology, Sydney, July 1999, and Conference on Risk Theory, Oberwolfach, Sep. 1999. Available at *http://www.math.ethz.ch/ schmock/ftp/slides/AllocationSlidesOct1999.pdf.*

[19] W.F. Sharpe. Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance* (Sep. 1964), 425-442.

[20] D. Tasche. Risk contributions and performance measurement. Technical document, Munich University of Technology, 1999. Available at *http://citeseer.nj.nec.com/tasche99risk.html.*

[21] D. Tasche. Expected shortfall and beyond. *Journal of Banking and Finance* 26, 1519-1533, 2002.