Maturity as a factor for credit risk capital

Michael Kalkbrener*, Ludger Overbeck †

Deutsche Bank AG, Corporate & Investment Bank, Credit Risk Management

1 Introduction

1.1 Quantification of maturity effects

In this paper we analyse maturity effects on the risk capital for credit portfolios. Conceptually, we deal with the question how the risk capital of a loan or bond with maturity m_1, e.g. $= 3$ years, differs from the risk capital of a loan with maturity m_2, e.g. 7 years. These maturity factors are important in light of the Basel II discussion [2]. Since internal credit risk models are not accepted, regulatory authorities suggest capturing maturity effects in terms of multipliers. Basel II allows measurement of the risk of a standard asset (at the moment with maturity 3 years) by internal ratings. The impact of other maturities should then be expressed in terms of fixed multipliers applied to the capital of the 3 year asset. The present paper attempts to give some insight in the derivation and size of these factors. In general our adjustments are lower than in the MTM approach of the current Basel II proposal.

To be consistent with industry standards and the formal derivation of the risk weight function in the Basel II consultation paper we assume that risk capital is based on a planning horizon of 1 year. Our setting for computing risk capital is similar to the Credit Metrics^{TM} resp. KMV approach [6, 15, 8, 12]. The rating or creditworthiness of all counterparties at year one is determined by an underlying multivariate variable $\vec{A}$ which might be called "Asset-Value-Process" or more generally "Ability-to-Pay" process. A loss distribution of the credit portfolio is computed by revaluation based on the "Ability-to-Pay" process and the maturity structure of the portfolio. The risk capital of the portfolio corresponds to a quantile of its loss distribution. Details of this model are given in section 2.

We present two different approaches to analyse maturity effects. The first one, called "one particle approach", is based on a notion of contributory capital of an individual credit in a portfolio. The second approach considers how the capital of an entire portfolio changes if the maturity of all loans in the portfolio change.

*Michael.Kalkbrenner@db.com

†Ludger.Overbeck@db.com

The views expressed in this note are those of the authors and do not necessarily reflect the position of Deutsche Bank AG.

In the ”one particle approach”, we construct a diversified portfolio of loans with different ratings and maturities. We add one loan C and compute its contributory economic capital E_1. Then the maturity of the loan is changed and its contributory economic capital E_2 is computed again. The quotient between E_1 and E_2 measures the maturity effect for the rating class of C. In section 3, this analysis is done for all rating classes.

We use two definitions of contributory capital, namely one based on the covariance of a loan with the portfolio and one based on contributory expected shortfall, i.e. the average contribution of a loan to very large portfolio losses. Both capital allocation techniques are based on Monte Carlo simulation. The covariance approach is independent of the quantile chosen for the definition of capital and is - in our opinion - not suitable for the calculation of maturity adjustments. In contrast, contributory expected shortfall is sensitive to the quantile. It turns out that with higher quantiles, i.e. if capital is defined in terms of extreme risk, the influence of maturity decreases significantly.

The second approach, which is presented in section 4, focuses on portfolios which consist of loans of the same credit quality and maturity. Capital is defined as the Credit-VaR of the entire portfolio. Maturity adjustments are determined by varying the maturity of the portfolio and computing corresponding changes in portfolio capital. We use Monte Carlo simulation to compute maturity adjustments for portfolios of different rating and size and give evidence that adjustments converge if portfolio size increases. The limits are the maturity adjustments for infinite portfolios which we compute by an analytic generalization of the risk weight function BRW in the Basel proposal [2]. In general, maturity adjustments obtained by this homogeneous portfolio approach are similar to those based on contributory expected shortfall in the ”one particle approach”.

1.2 Main results

Maturity effects increase with credit quality, i.e. higher ratings have higher maturity adjustments than lower ratings. This qualitative result can easily be verified by each of the estimation techniques used in this note. It is also in line with the adjustments proposed in the Basel II consultation paper. However, the quantification of maturity effects is a more challenging task. One problem is that maturity adjustments heavily depend on estimation techniques and parameter settings. In this paper, we experiment with the following parameters:

1. *Migration matrices:* We use the one-year migration matrix presented by S&P in [14], the KMV matrix in [9] and a matrix GC constructed from migration data on German corporates (see appendix 1). In all tests the choice of the migration matrix is critical. The KMV matrix produces the highest and the S&P matrix the lowest maturity adjustments.

2. *Spreads:* The revaluation of the portfolio at the end of the 1y planning period is based on credit spreads resp. corresponding multi-year default probabilities. The spreads are either market spreads or derived from migration matrices (and therefore based on historical data). In spite of the fact that a proper mark-to-market of traded credit products has to be based on market spreads we think that historical spreads have a number of advantages for the analysis of maturity effects: historical spreads are less volatile and they

do not reflect liquidity risk and risk aversion (including the cost of risk capital which we intend to derive). Another argument against market spreads is the fact that most loans are not liquid assets.

In our analysis we use historical spreads as well as market spreads (see appendix 2). Results show that maturity adjustments are sensitive to the choice of spreads. In particular, the comparatively high spreads in September 2001 lead to higher adjustments.

3. *Quantiles:* Expected shortfall contributions for single exposures and Credit-VaR for portfolios are defined with respect to specific quantiles. We show that maturity effects rapidly decrease if higher quantiles are considered.

4. *Other parameters:* We experiment with different government yield curves, recovery assumptions, one- and multi-factor correlation models and different average asset correlations. The variations in results are minor compared to matrix, spread and quantile effects.

We use different estimation techniques for maturity adjustments:

1. Although the var/covar technique has obvious disadvantages if applied to fat tail distributions it is the standard technique for allocating credit EC. We therefore use var/covar allocation in the one-particle approach: the **one particle approach based on covariance** produces maturity adjustments which are significantly higher than those computed with other techniques. The results are independent of the quantile chosen for the definition of capital.

2. The **one particle approach based on shortfall contributions** and the **homogeneous portfolio approach** give rather consistent results. For instance, we obtain the following factors between a 1 year facility and a 7 year facility for the best rating class[1] with the GC matrix (see sections 3.3.3 and 4.2): 2.68 resp. 2.22 for the 99.9065% resp. 99.98% quantile with the one-particle-approach and 2.43 resp. 1.97 with the portfolio approach.

Our results show that the impact of maturity decreases if the confidence level for capital is increased. This is consistent with the evidence that extreme loss events in credit risk are predominately caused by defaults, cf. Chapter 11 in [6]. Our findings give therefore evidence that at least for regulatory purposes, which should focus on systemic extreme risk, the adjustments in the MTM approach of the current Basel II consultation paper are too high. Based on the results in the present paper our recommendation is to **cap maturity adjustments between one and seven years at 2.5**. In our opinion, higher adjustments would lead to a misallocation of capital, namely against the volatility of migration and not against extreme losses.

2 Basic model

The computation of maturity adjustments is based on risk capital and risk contributions, which are derived from the underlying loss variables of the portfolio and individual exposures. This

[1] The best rating class has a one-year default probability of 3bp.

section presents the basic model.

2.1 Loss variables

Each loan C_i in the portfolio has a loss variable L_i, which specifies the value of the loan at the planning horizon of one year. The portfolio loss is defined by the random variable $L = \sum_{i=1}^{N} L_i$, where N is the number of facilities in the portfolio.

2.2 Ability-to-Pay

Let $\vec{A}$ be an N-variate standardized[2] normally distributed random variable with correlation matrix R. We call the i-th component of $\vec{A}$ the "Ability-to-Pay" of loan C_i. The general model is $L_i = L(A_i, m_i)$, i.e. the value resp. the loss function of the loan depends solely on the ability to pay of the counterparty and the maturity m_i of the loan[3]. In the KMV concept L is a continuous function of A_i. For simplicity, we assume in this note that it is a step function, i.e. only a finite number of values can be obtained. In credit risk modeling it is common to identify these finitely many states with credit ratings.

2.3 Ratings and transition matrices

We use the S&P rating scheme consisting of the rating classes AAA, AA, A, BBB, BB, B and CCC. These ratings are identified with numbers $1, \ldots, 7$ and the additional rating class Default with 8. A one-year migration matrix $M = (p_{ij})_{i,j=1,..,8}$ specifies the probabilities p_{ij} that a company migrates from rating i to rating j in a one-year period. We use three different transition matrices for our analysis: the S&P matrix in [14], the KMV matrix in Kealhofer et al. [9] and a transition matrix GC derived from migration data on German corporates. The exact definition of these three matrices is given in appendix 1.

The values of the "Ability-to-Pay" processes $A_1, \ldots, A_N$ in one year determine the ratings of the loans: the rating migration is simulated by defining thresholds $D_{k,i}$ in the distribution of the A_i such that the event "counterparty i migrates to rating k" coincides with the event "the value of A_i in one year lies between $D_{k,i}$ and $D_{k+1,i}$".

2.4 Revaluation techniques and spreads

We assume that each facility C_i in the portfolio has the cashflow profile of a bullet bond at par. Each C_i is revaluated under the assumption that it is rated $k = 1, \ldots, 8$ in one year, yielding eight different values $V_1, \ldots, V_8$ of C_i. The revaluation formula is based on government bond yields and multi-year default probabilities which are derived from transition matrices[4] or cor-

[2] This means that all standard deviations are 1.

[3] We assume that all loans have the same valuation function, i.e. have the same product specification but only different maturities.

[4] Note that the last column of each transition matrix M specifies the one year default probabilities for all rating classes. Under the assumption that the rating process is a time-homogeneous Markov process the t-year default probabilities can be obtained from the last column of the t-th power of M.

porate bond spreads (see appendix 2 for the specification of spreads used in this paper). The loss variable L_i is defined by subtracting the vector $(V_1, \dots, V_8)$ from the value $\bar{V}$ of C_i if its current rating has not changed.

2.5 Economic capital

The economic capital of the portfolio is either defined as a quantile of the loss variable L or as a quantile minus the mean of L. We have used both definitions in our analysis and have not found a significant impact on maturity adjustments.

3 One particle approach

In this approach we construct a diversified portfolio with different ratings and maturities. We add one loan C and compute its contributory economic capital E_1. In the next step the maturity of the loan is changed and its contributory economic capital E_2 is computed again. The quotient between E_1 and E_2 measures the maturity effect for the rating class of C.

In subsection 3.1 the portfolio is constructed. Subsection 3.2 presents maturity adjustments based on the classical var/covar contribution technique. In subsection 3.3 expected shortfall contribution is introduced and applied to the computation of maturity adjustments.

3.1 Construction of a diversified portfolio

Let $\{0.03, 0.05, 0.1, 0.2, 1, 3.3, 15\}$ be a set of default probabilities (in percent) and $\{1, 2, 3, 4, 5, 6, 7\}$ a set of maturities (in years). We consider a portfolio P which consists of 98 loans, each possible default probability and maturity combination appearing twice. Now we add a single loan $C_{p,m}$ to the portfolio with default probability $p \in \{0.03, 0.05, 0.1, 0.2, 1, 3.3, 15\}$ and maturity $m \in \{1, 3, 7\}$. In this way 21 different portfolios $P_{p,m} := P \cup \{C_{p,m}\}$ are obtained. It is assumed that all loans have the same notional. In the initial test scenario the recovery rate of each loan is 50% and the correlation structure is specified by a one-factor model with all asset correlations equal to 35%.

3.2 Contributory EC based on var/covar

The var/covar contribution technique is the standard approach developed in the capital asset pricing model. Risk contributions are proportional to covariances of loss variables of individual loans and portfolios. Since we are not interested in absolute contributory EC numbers but only in ratios of contributory economic capital we proceed in the following way: for each $p \in \{0.03, 0.05, 0.1, 0.2, 1, 3.3, 15\}$ and $m \in \{1, 3, 7\}$ the loss variables of loan $C_{p,m}$ and portfolio $P_{p,m}$ are simulated[5] and the covariance $cov_{p,m}$ is computed. In accordance with the definition of maturity adjustments in the Basel proposal [2] risk contributions are normalized

[5] Our results are based on 400.000 Monte Carlo simulations.

at 3 years, i.e. we compute $cov_{p,m}/cov_{p,3}$. Note that the results are independent of the quantile chosen for the capital definition.

3.2.1 Test: GC matrix

We derive migration probabilities and multi-year default probabilities from the GC matrix and obtain

	1	3	7
0.03	0.42	1	1.77
0.05	0.45	1	1.73
0.10	0.57	1	1.58
0.20	0.74	1	1.37
1.00	0.75	1	1.23
3.30	0.83	1	1.13
15.00	0.97	1	1.03

This result table is structured in the following way. The rows correspond to the different default probabilities 0.03, 0.05, 0.1, 0.2, 1, 3.3 and 15 percent and the columns correspond to the considered maturities 1, 3, and 7 years. Since the table is normalised to the 3 year capital all entries in the second column are 1. As an example, the first entry ”0.42” in the first row means that the capital for a 1 year deal with default probability of 0.03 is 42% of the capital for a 3 year deal with the same default probability.

Remark: Since not all of the default probabilities 0.03, 0.05, 0.1, 0.2, 1, 3.3 and 15 percent correspond to GC ratings the transition probabilities of some loans are computed by interpolation in the GC matrix. For instance, the transition probabilities for a loan with PD=0.1 are obtained by linear interpolation between the transition probabilities of the rating class 1 (PD=0.07) and rating class 2 (PD=0.2).

The above table shows that maturity effects increase with credit quality. This qualitative result has been verified with each of the estimation techniques used in this note. In the following we will therefore focus on adjustments for the senior rating class (PD=0.03). Full results for all rating classes are presented in an extended version of this paper (see Kalkbrener and Overbeck [7]).

3.2.2 Tests: S&P resp. KMV matrix

We repeat the above test with the same portfolio but use the S&P resp. KMV matrix instead of the German corporates. The following table displays the maturity adjustments for the senior rating class obtained with each of these three matrices:

GC			**S&P**			**KMV**		
1	3	7	1	3	7	1	3	7
0.42	1	1.77	0.57	1	1.67	0.29	1	2.80

The differences are significant. The factors between 1 and 7 years are 4.21 for the GC matrix, 2.93 for the S&P matrix and 9.66 for the KMV matrix. Basically, there are two main reasons for these differences.

1. Migration volatility: Migration volatility can be read off the diagonal of the migration matrix. KMV postulates a very high migration probability. For instance, only 66% remain in the best rating class compared to 91% in the S&P matrix. Higher migration probabilities induce higher capital requirements.

2. Revaluation: In our setting the revaluation is driven by the last column of the migration matrix. The impact of migration varies since the differences in the default probabilities of different ratings are important. The steepest gradient can be found in the S&P matrix. The default probabilities range from 0.01% to 20%, whereas KMV only covers 0.02% to 10.13%. The GC matrix shows the most stable repricing, the range is from 0.07% to 6%.

Obviously, the high maturity adjustments obtained with the KMV matrix are caused by its high migration volatility which is not fully compensated by its less volatile repricing (compared to S&P).

3.2.3 Tests: S&P with market spreads

In the previous tests revaluation was based on multi-year default probabilities obtained from historical transition matrices. In this subsection multi-year default probabilities are derived from the corporate bond spreads in appendix 2. Since these bond spreads are based on the S&P rating system we use the S&P transition matrix for the specification of the 1y migration probabilities:

Spreads 97			**Spreads 01**		
1	3	7	1	3	7
0.55	1	1.92	0.34	1	2.59

Note that the comparatively high spreads on 25th September 2001 lead to higher adjustments.

3.2.4 Additional tests

We did additional tests with the GC matrix by

1. using different government yield curves,

2. changing the recovery rate to 70%,

3. varying average asset correlations between 20% and 50%,

4. replacing the one-factor model by a ten-factor model[6] and

[6]Factor loadings are randomly generated such that average correlation is close to 35%.

5. using randomly generated portfolios.[7]

Our results show that these changes have little effect compared to differences caused by using different transition matrices and spreads.

3.3 Contributory EC based on expected shortfall

3.3.1 Alternative economic capital definition

From a risk management point of view, holding the economic capital based on a quantile, say the 99.5% quantile, as cushion against the portfolio means that in average in 199 out of 200 years the capital would cover all losses. The disadvantage of this definition is that it does not take the size of the losses in the extreme 0.5% tail into account. Hence, this approach towards economic capital resembles an ”all or nothing” rule. In particular, in a ”bad” year (1 out of 200) the capital does not cushion the losses. An alternative to EC based on quantiles is the following capital definition which focuses on large portfolio losses: consider those losses which exceed a given amount K and let economic capital (based on shortfall) be defined by

$$\mathrm{EC}_K(S) \quad := \quad E[L|L > K].$$

Hence, economic capital based on shortfall covers the average ”bad” loss. This approach also motivates the following definition of contributory capital based on coherent risk measures.

Coherency is analysed in detail by Artzner et al. [1] and Delbaen [3]. They show that for continuous distributions, $EC_K(S)$ is coherent if K is a quantile of L. Coherency requires a risk measure to satisfy a set of axioms or first principles that a reasonable risk measure should obey. These axioms include sublinearity. It is also shown that the risk measures defined in terms of quantiles are not coherent in general.

3.3.2 Risk contributions

An important advantage of $\mathrm{EC}_K(S)$ is the simple allocation of risk capital to a single transaction, cf.[13]. The contribution to shortfall risk, CSR, or shortfall contribution is defined by

$$CSR_i \quad = \quad E[L_i|L > K].$$

That is, the capital for a single loan is its average loss in bad years. Hence, a capital quota of over 100% is impossible, in contrast to the classical var/covar approach.

[7]Portfolios consist of 98 loans. Possible PDs are 0.03, 0.05, 0.1, 0.2, 1, 3.3 and 15 percent and maturities 1, 2, 3, 4, 5, 6, 7 years. Exposure size varies between 10 and 100. Exposures and maturities are uniformly distributed in each rating class.

3.3.3 Tests with different matrices and spreads

We compute the shortfall contributions of the loans $C_{p,m}$ in the portfolios $P_{p,m}$. The threshold K is defined by $E[L|L > K] = \alpha$-quantile, where $\alpha = 99.9065\%$. The following table displays the results obtained with all three matrices and two sets of market spreads.

GC			**S&P**			**KMV**			**Spreads 97**			**Spreads 01**		
1	3	7	1	3	7	1	3	7	1	3	7	1	3	7
0.56	1	1.50	0.69	1	1.39	0.54	1	1.73	0.67	1	1.75	0.61	1	1.87

These maturity adjustments are considerably smaller than those implied by the standard allocation technique based on covariances. For instance, the KMV factor between 1 and 7 years for the senior rating decreased from 9.66 to 3.2. This result is not surprising from an economic point of view. Migration increases volatility but not necessarily ”extreme” or ”tail” risk which is the basis for expected shortfall and also the main concern of regulators. This becomes particularly obvious for the KMV matrix which has high migration volatility.

In order to analyse the quantile effect computations are repeated for the 99.5% and 99.98% quantiles. The following table compares factors between 1 and 7 years for the senior rating.

	GC	**S&P**	**KMV**	**S’97**	**S’01**
var/covar	4.21	2.93	9.66	3.49	7.62
99.50%	2.94	2.16	4.35	2.92	5.40
99.91%	2.68	2.01	3.20	2.61	3.07
99.98%	2.22	1.74	2.60	1.74	2.36

The structure of results is consistent across transition matrices and spreads: the highest factors are obtained in the var/covar approach, factors significantly decrease if higher quantiles are considered (between 1.74 and 2.6 for the 99.98% quantile).

4 Portfolio approach

4.1 Construction of homogeneous portfolios

In this section we consider portfolios which are not diversified in terms of maturities, i.e. all loans in the portfolio have the same maturity. Furthermore, we make the additional assumption that all loans in a portfolio have the same rating. Of course, this is not a realistic assumption but similar to the one used in the Basel proposal [2]. In this proposal the regulatory capital charge of a single loan with default probability p equals the quantile of a percentage loss distribution of an infinitely large homogeneous portfolio with default probability p and asset correlation 20%. This is consistent with a one-factor model for an infinite granular portfolio. Since the one-factor model is portfolio invariant there is no differentiation between different portfolios, in particular it is not sensitive to diversification efforts.

In the following analysis we consider 21 homogeneous portfolios $P_{p,m}$, $p \in \{0.03, 0.05,$

$0.1, 0.2, 1, 3.3, 15\}$ and $m \in \{1, 3, 7\}$, each consisting of 100 loans with default probability p and maturity m. It is assumed that the correlation structure is specified by a one-factor model with all asset correlations equal to ρ. Capital is defined as the Credit-VaR of the entire portfolio. The maturity effect is determined by varying the maturity of the portfolio and computing corresponding changes in portfolio capital. [8]

4.2 Tests with portfolios of different size

The first three numbers 0.64, 1, 1.37 in the result table

100 loans			400 loans			∞ loans		
1	3	7	1	3	7	1	3	7
0.64	1	1.37	0.61	1	1.44	0.60	1	1.46

are the ratios

$$q\text{-quantile}(P_{p,m}) \,/\, q\text{-quantile}(P_{p,3}) \qquad m = 1, 3, 7$$

for $q = 99.9065\%$ and the senior rating $p = 0.03$. The asset correlation $\rho = 35\%$ is used. Rating migration and multi-year default probabilities are specified by the GC matrix. The computations are repeated for portfolios consisting of 400 loans. The last three numbers correspond to portfolios with an infinite number of loans. For these infinite portfolios the portfolio loss variable equals (see [4], [10] or [7])

$$L_i(1) + \sum_{j=1}^{7} (L_i(j+1) - L_i(j)) \cdot N\left((c_j - \sqrt{\rho}\, Y) \,/\, \sqrt{1-\rho}\right), \tag{1}$$

where $L_i(1), \ldots, L_i(8)$ resp. $p_i(1), \ldots, p_i(8)$ specify the loss function resp. the one-year migration probabilities of the individual loans C_i,[9] Y is a standard normally distributed variable, N denotes the standard normal distribution function and

$$c_j := N^{-1}\left(\sum_{l=1+j}^{8} p_i(l)\right). \tag{2}$$

Since the correlation structure is specified by a one-factor model the Credit-VaR can be computed analytically. In contrast, the results for the finite portfolios have been obtained by 400.000 Monte Carlo simulations.

As expected, maturity adjustments converge to the limit specified by the infinite portfolio. Note that the results are similar to those obtained by shortfall contribution in section 3.3.3. This similarity can also be observed for the other matrices, spreads and quantiles. The following table displays factors between 1 and 7 years for the senior rating computed with analytic

[8] Note that in this model the contributory capital of each loan equals the total risk capital divided by the number of loans regardless of the capital allocation technique used.

[9] As defined in section 2, the 7 non-default rating classes correspond to $1, \ldots, 7$ and default corresponds to 8.

formula (1). Note that these adjustments have the same magnitude as the maturity adjustments based on contributory expected shortfall in the ”one particle approach” (last table in section 3.3.3).

	GC	**S&P**	**KMV**	**S’97**	**S’01**
99.50%	3.46	2.57	6.10	2.46	5.41
99.91%	2.43	2.09	3.73	1.95	3.54
99.98%	1.97	1.82	2.72	1.69	2.69

5 Conclusion

For regulatory purposes the required capital is formally based on a 99.9065% quantile. Taking all add-ons into account actual capital requirements correspond to a much higher quantile. The results in this paper support the view that on this extreme security level, migration and therefore maturity are of minor importance. Maturity adjustments computed with methods sensitive to quantiles are significantly lower than adjustments obtained with the classical var/covar contribution technique. Our results show that if capital requirements are based on high quantiles a maturity adjustment factor of 2.5 between 1 year and 7 years is a conservative setting even for the best rating classes.

References

[1] P. Artzner, F. Delbaen, J.-M. Eber and D. Heath. Coherent measures of risk. *Mathematical Finance* 9, 203-228, 1999.

[2] Basel Committee on Banking Supervision. The internal ratings-based approach. Consultative Document, Basel, January 2001.

[3] F. Delbaen. Coherent risk measures. Lecture notes, Scuola Normale Superiore di Pisa, 2001.

[4] C. Finger. Conditional approaches for CreditMetrics portfolio distribution. *CreditMetrics Monitor* 1, 14-33, April 1999.

[5] M. Gordy and E. Heitfield. Maturity effects in a class of multi-period default mode models. Board of Governors of the Federal Reserve System, June 2001.

[6] J.P. Morgan. *Credit Metrics*. Technical Document, 1997.

[7] M. Kalkbrener and L. Overbeck. Maturity as a factor for credit risk capital. Technical Document, CIB/CRM/RAI Risk Research & Development, Deutsche Bank AG, Frankfurt, 2001.

[8] S. Kealhofer. Managing default risk in derivative portfolios. *Derivative Credit Risk: Advances in Measurement and Management*, Renaissance Risk Publications, 1995.

[9] S. Kealhofer, S. Kwok and W. Weng. Uses and abuses of bond default rates. KMV Corporation, San Francisco, 1998.

[10] A. Lucas, P. Klaassen, P. Spreij and S. Straetmans. An analytic approach to credit risk of large corporate bond and loan portfolios. To appear in the *Journal of Banking and Finance*, 2001.

[11] A. Machauer and M. Weber. Bank behavior based on internal credit ratings of borrowers. Working paper 98/08, Center for Financial Studies, Frankfurt, 1998.

[12] L. Overbeck and G. Stahl. Stochastic models in risk managing a credit portfolio. To appear in *Kredit und Kapital*, 2001.

[13] L. Overbeck. Allocation of economic capital in loan portfolios. Proceedings ”Measuring Risk in Complex Stochastic Systems”, Berlin 1999. Editors: Stahl/Härdle, *Lecture Notes in Statistics*, Springer, 2000.

[14] Standard & Poor’s. Standard & Poor’s Rating Performance 1998. New York, 1999.

[15] O. A. Vasicek. Credit valuation. *Net Exposure* 1, October 1997.

Appendix 1: transition matrices

We use three different transition matrices for our analysis. The following transition matrix is based on S&P [14]:

	AAA	AA	A	BBB	BB	B	CCC	$Def.$
AAA	91.39	7.91	0.52	0.08	0.04	0.03	0.02	0.01
AA	0.72	91.62	6.77	0.64	0.07	0.12	0.03	0.03
A	0.08	2.40	91.05	5.45	0.66	0.28	0.01	0.07
BBB	0.05	0.30	6.03	86.66	5.36	1.22	0.18	0.20
BB	0.02	0.13	0.66	7.49	80.78	8.86	1.04	1.02
B	0.00	0.08	0.35	0.50	6.67	83.56	3.68	5.16
CCC	0.13	0.00	0.34	0.69	1.71	12.50	64.63	20.00
$Def.$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Kealhofer et al. [9] question that rating changes are a good indicator for credit quality changes. In particular, they claim that rating agencies are too slow in changing ratings and therefore the probability of staying in a grade overstates the true probability to keep approximately the same credit quality. Therefore, the following approach based on KMV’s expected default frequencies (EDFs) is proposed. Firms are classified based upon non-overlapping ranges of default probabilities. Each of these ranges corresponds to a rating class, i.e. firms with default rates less than or equal to 0.02% are in *AAA*, 0.03% to 0.06% corresponds to *AA*, etc. The historical frequencies of changes from one range to another are computed from the history of changes in default rates as measured by EDFs. This gives the following KMV one-year transition matrix:

	AAA	AA	A	BBB	BB	B	CCC	$Def.$
AAA	66.26	22.22	7.37	2.45	0.86	0.67	0.15	0.02
AA	21.66	43.04	25.83	6.56	1.99	0.68	0.20	0.04
A	2.76	20.34	44.19	22.94	7.42	1.97	0.28	0.10
BBB	0.30	2.80	22.63	42.54	23.52	6.95	1.00	0.26
BB	0.08	0.24	3.69	22.93	44.41	24.53	3.41	0.71
B	0.01	0.05	0.39	3.48	20.47	53.01	20.58	2.01
CCC	0.00	0.01	0.09	0.26	1.79	17.77	69.95	10.13
$Def.$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

The third transition matrix, abbreviated GC matrix, is derived from migration data on German corporates (see also [11])

	AAA	AA	A	BBB	BB	B	CCC	$Def.$
AAA	71.55	19.86	5.43	1.90	0.24	0.37	0.58	0.07
AA	2.18	71.06	21.14	4.14	0.70	0.29	0.29	0.20
A	0.18	6.43	69.72	19.47	2.54	0.80	0.51	0.35
BBB	0.06	1.10	16.43	64.74	11.26	3.84	1.77	0.80
BB	0.07	0.64	5.39	27.86	43.23	14.34	6.87	1.60
B	0.02	0.42	3.12	12.95	16.48	45.46	18.55	3.00
CCC	0.16	0.61	2.36	3.75	4.26	8.67	74.19	6.00
$Def.$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Appendix 2: corporate bond spreads

We use the following corporate bond spreads under normal and distressed market conditions:

	Spreads 97					**Spreads 01**				
	1y	2y	3y	5y	7y	1y	2y	3y	5y	7y
AAA	0.16	0.18	0.22	0.25	0.31	0.40	0.45	0.50	0.60	0.74
AA	0.20	0.22	0.26	0.30	0.35	0.53	0.60	0.65	0.76	0.90
A	0.27	0.30	0.32	0.37	0.42	0.80	0.90	1.01	1.18	1.36
BBB	0.44	0.46	0.50	0.52	0.56	1.21	1.30	1.41	1.59	1.79
BB	0.89	1.06	1.20	1.41	1.59	2.58	2.91	3.16	3.50	3.73
B	1.50	1.63	1.83	2.11	2.37	4.41	4.83	5.41	6.25	6.98
CCC	2.55	3.00	4.00	5.00	6.00	6.00	6.50	8.00	9.00	10.25

Spreads 97 (resp. Spreads 01) are the spreads of US industrial bonds over government yields on July 11, 1997 (resp. Sep. 25, 2001). The rating system is S&P. The data source is file SPRDCRV.TXT from CreditMetrics. Spreads 97 have also been used in [5].