Risk management of non-maturing liabilities

Michael Kalkbrener* Jan Willing†

Abstract

Risk management of non-maturing liabilities is a relatively unstudied issue of significant practical importance. Non-maturing liabilities include most of the traditional deposit accounts like demand deposits, savings accounts and short time deposits and form the basis of the funding of depository institutions. Therefore, the asset and liability management of depository institutions depends crucially on an accurate understanding of the liquidity risk and interest rate risk profile of these deposits.

In this paper we propose a stochastic three-factor model as general quantitative framework for liquidity risk and interest rate risk management for non-maturing liabilities. It consists of three building blocks: market rates, deposit rates and deposit volumes. We give a detailed model specification and present algorithms for simulation and calibration. Our approach to liquidity risk management is based on the term structure of liquidity, a concept which forecasts for a specified period and probability what amount of cash is available for investment. For interest rate risk management we compute the value, the risk profile and the replicating bond portfolio of non-maturing liabilities using arbitrage-free pricing under a variance-minimizing measure. The proposed methodology is demonstrated by means of a case study: the risk management of savings accounts.

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*Deutsche Bank AG, CIB/Credit Risk Management, Risk Analytics & Instruments

†Münchener Rückversicherungs-Gesellschaft

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1 Introduction

1.1 Importance of non-maturing liabilities in the AL management

The asset/liability problem that depository institutions face is quite simple to explain - although not necessarily easy to solve. A depository institution seeks to earn positive spread between the assets it invests in (loans and securities) and the cost of its funds (deposits and other sources). The spread income should allow the institution to meet operating expenses and earn fair profit on its capital.

In generating spread income a depository institution faces several risks, most important credit risk, interest rate (or funding) risk and liquidity risk. The management of interest rate and liquidity risk is particularly difficult for non-maturing liabilities, i.e. deposits without a specific maturity or deposits whose actual time horizon significantly differs from their contractual maturity. Non-maturing liabilities include most of the traditional deposit accounts like demand deposits, savings accounts and short time deposits and form the basis of the funding of depository institutions. Therefore, the asset and liability management of depository institutions depends crucially on an accurate understanding of the liquidity risk and interest rate risk profile of these deposits.

For quite some time bank regulators have been aware of the significance of proper risk management for non-maturing liabilities but also of the difficulties. In fact, uncertainty about measuring interest rate risk for these deposits was one factor in not adopting formal interest rate capital guidelines for banks' non trading positions (see Federal Reserve (1995) or O'Brien (2000)). Recently several consulting papers proposed the introduction of a capital charge for institutions with a high exposure to interest rate risk arising from banking book items like non-maturing deposits (BIS, 1999; EU, 1999). These new regulations will increase the pressure on depository institutions to use transparent quantitative methodologies for the risk management of these positions.

1.2 Quantitative risk management techniques

The central problem in the risk management of non-maturing liabilities is the assignment of a maturity profile to these liabilities or, equivalently, the construction of a replicating bond portfolio with fixed maturities. Many banks use the following approach:

1. First the core of a balance position is determined, then the floating part is defined as balance minus core.

2. The floating part is invested in the overnight (O/N) bucket. The core is subdivided into portions which are invested in different time bands. Maturing tranches are reinvested in the same time band.

The subdivision into floating part and core portions with different maturities is usually done in a rather arbitrary way without theoretical justification. In fact, despite its significance this problem is still relatively unstudied.

In the last ten years a number of papers on managing non-maturing assets and liabilities have been published including Ausubel (1991), O'Brien et al. (1994), the Office of Thrift Supervision (1994), Hutchinson and Pennacchi (1996), Selvaggio (1996), Jarrow and van Deventer (1998), Schürle (1998), Janosi et al. (1999) and O'Brien (2000). Hutchinson and Pennacchi (1996) value non-maturing liabilities in an equilibrium-based model. Jarrow and van Deventer (1998) propose an arbitrage-free approach for valuation and hedging. They show that non-maturing liabilities are equivalent to particular interest rate swaps. They obtain an analytic valuation formula in a simple one-factor model with deposit rates and volumes given by deterministic functions of the short rate and the short rate specified by an extended one-factor Vasicek model. Janosi et al. (1999) provide an empirical investigation of the Jarrow and van Deventer model in the US market. O'Brien (2000) focuses on the US market as well. In his valuation model deposit rates and balances are represented by autoregressive processes. Alternative deposit rate specifications studied include asymmetric adjustment to market rate changes. Schürle (1998) applies stochastic optimization techniques to the margin optimization problem for non-maturing liabilities (see also Frauendorfer and Schürle, 2000).

The objective of our paper is the development of a general quantitative framework for liquidity risk and interest rate risk management for non-maturing liabilities. Our model consists of three building blocks.

Market rates: The market rate model serves as framework for valuing the cash flows of the non-maturing liabilities specified by deposit rates and volumes. In contrast to derivative pricing, our main focus is a realistic development of interest rates over a long period of time and not the exact fit to current market prices of plain vanilla instruments. We therefore use historical time-series for calibration. We have currently implemented two classes of two-factor HJM models: two-factor Vasicek models and non-parametric HJM models with piecewise constant volatility functions[1]. The calibration of both models is based on principal component analysis. In particular for the non-parametric model we have obtained very good results (Kalkbrener, 2002).

Deposit rates: Deposit rates are heavily influenced by market rates but rates of different types of deposits differ significantly. In particular, sensitivities to interest rate changes vary. We therefore propose the following general concept for modeling deposit rates: deposit rates are given by a deterministic function with only the market rates as stochastic arguments, no additional stochastic factor is used. Market rates of different maturities can be used as arguments and - even more important - no restrictions are made concerning the form of the deterministic function.

[1]All models and algorithms presented in this paper have been implemented in the computer algebra system *Mathematica* (Wolfram, 1991).

Deposit volumes: Our analysis shows that correlations between deposit volumes and market rates are not particularly high in the German market. We therefore introduce an additional stochastic factor for deposit volumes. This factor may be correlated to the two factors of the market rate model. We have currently implemented two diffusion models for deposit volumes, a normally and a lognormally distributed model.

By combining the three components we obtain a three-factor model for non-maturing liabilities which we use as framework for risk management.

Liquidity risk management: Because of their high volumes and their characteristics non-maturing liabilities are of great importance for the liquidity risk management in depository institutions. The central problem is the following: what amount of cash is available for investment over a given time horizon $[0, t]$ with a given probability p? In order to answer this question we introduce the following simple concept. Let $V(u)$ be a stochastic process which specifies the volume of deposit accounts and define the process of minima by

$$M(t) := \min_{0 \leq u \leq t} V(u).$$

Denote the p-quantile of $M(t)$ by $TSL(t, p)$. Note that the probability that the volume drops below level $TSL(t, p)$ in the time interval $[0, t]$ is exactly p. We call the function TSL the term structure of liquidity. We use this concept to compute liquidity restrictions and to immunize a replicating portfolio against liquidity risk. The term structure of liquidity is constructed by Monte Carlo simulation of the volume process $V(u)$.

Interest rate risk management: We follow Jarrow and van Deventer (1998) and use an arbitrage-free approach for valuation and hedging.

1. Pricing: Since deposit rates are typically lower than market rates on comparable (equal risk) financial securities these deposits have a positive value for financial institutions. In order to compute this price we first have to apply a measure transformation from the real-world measure, which has been used for calibration, to a risk-neutral measure. In our model the risk-neutral measure is not uniquely defined because of model incompleteness caused by the introduction of the additional stochastic factor for deposit volumes. We use the concept of variance-minimizing measures for identifying a specific risk-neutral measure.

2. Portfolio replication: The pricing methodology is applied to the computation of sensitivities to different input parameters. In particular, deltas are computed by shifting the yield curve resp. parts of the yield curve. The resulting delta profile is used for constructing a replicating bond portfolio. If certain deposits show significant vega risk our methodology can be easily extended to portfolios

which contain not only bonds but also caps and floors (Ho, 1992; Ho and Chen, 1995).

One important advantage of arbitrage-free pricing for non-maturing liabilities is its consistency with the current practice of derivatives pricing and with the risk management methods for the trading book. Furthermore, our portfolio replication methodology is in line with the duration based approach suggested by the European Commission (EU, 1999): *The Commission services propose the duration-based method (which requires the measurement of the sensitivity of positions to changes in yield) ... as the standard methodology for assessing interest rate risk in the banking book.*

The rest of the paper is structured in the following way. In section 2 our three factor model for non-maturing liabilities is specified. After giving a few stochastic preliminaries in section 2.1, the market rate model is defined in section 2.2. In particular, a formula for simulating non-parametric HJM models is derived and our approach to parameter estimation is presented. Sections 2.3 and 2.4 specify the models for deposit rates and volumes. Section 3 is on the computation of liquidity forecasts and section 4 on the management of interest rate risk. In section 5 a case study on the risk management of savings accounts is presented. The paper finishes with a short discussion of future research topics in section 6.

2 A three-factor model for non-maturing liabilities

A model which specifies market rates and deposit rates and volumes is defined in a continuous-time framework.

2.1 Stochastic preliminaries

Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space and $W(t) = (W_1(t), W_2(t), W_3(t))$ a three-dimensional Brownian motion. We denote the first two components $(W_1(t), W_2(t))$ by $\bar{W}(t)$. Let $\mathcal{F}_t$ resp. $\bar{\mathcal{F}}_t$ be the usual augmentations of the filtrations generated by $W(t)$ and $\bar{W}(t)$ respectively.

We will now proceed in the following way. First a treasury securities market is defined. Treasuries only depend on $\bar{W}(t)$, they are specified within the HJM framework. We ensure completeness of the treasury securities market by assuming uniqueness of a martingale measure $\bar{\mathcal{Q}}$ on $\bar{\mathcal{F}}$. The third component $W_3(t)$ of the Brownian motion will be used to define volume processes. The introduction of this additional stochastic factor leads to market incompleteness. We identify a unique extension of $\bar{\mathcal{Q}}$ to the filtration $\mathcal{F}$ using the concept of variance-minimizing measures.

2.1.1 Treasury securities market

We consider a continuous-time economy with trading horizon $[0, \tau]$. In order to specify the term structure of interest rates we follow Heath et al. (1992) and assume that for every fixed $T \leq \tau$, the dynamics of the instantaneous forward rate $f(t, T)$ are given by

$$f(t,T) = f(0,T) + \int_0^t \mu(v,T)dv + \int_0^t \sigma(v,T)d\bar{W}(v) \qquad \text{for } 0 \leq t \leq T.$$

Zero-coupon bond prices at time t with maturity at date T are denoted by $P(t, T)$ and specified in terms of forward rates, i.e.

$$P(t,T) := \exp(-\int_t^T f(t,v)dv).$$

The short rate $r(t)$ and the money market account's value $B(t)$ are given by

$$r(t) \ := \ f(t,t) \ = \ f(0,t) + \int_0^t \mu(v,t)dv + \int_0^t \sigma(v,t)d\bar{W}(v) \tag{1}$$

and

$$B(t) := \exp(\int_0^t r(v)dv).$$

2.1.2 Martingale measures

To exclude arbitrage opportunities in the Treasury securities market, we assume that there exists a probability measure $\bar{\mathcal{Q}}$ on $\bar{\mathcal{F}}_\tau$ which is equivalent to the restriction $\bar{\mathcal{P}}$ of $\mathcal{P}$ and which is a martingale measure relative to the short-rate process r, i.e. $P(t, T)/B(t)$ is a $\bar{\mathcal{Q}}$-martingale for all $0 \leq T \leq \tau$. Furthermore, we assume that $\bar{\mathcal{Q}}$ is the unique equivalent martingale measure on $\bar{\mathcal{F}}_\tau$. By definition of the filtration $\bar{\mathcal{F}}_t$, $\bar{\mathcal{Q}}$ can be obtained from $\bar{\mathcal{P}}$ by means of a Girsanov transformation, i.e. the density of $\bar{\mathcal{Q}}$ w.r.t. $\bar{\mathcal{P}}$ is of the form

$$\bar{\eta} := \exp(\int_0^\tau \bar{\lambda}(t)d\bar{W}(t) - \frac{1}{2}\int_0^\tau |\bar{\lambda}(t)|^2 dt), \tag{2}$$

where $\bar{\lambda}(t) = (\lambda_1(t), \lambda_2(t))$ is a 2-dimensional process adapted to $\bar{\mathcal{F}}_t$. For the rest of the paper we will assume that λ_1 and λ_2 are constants. These two parameters specify how the real world measure $\bar{\mathcal{P}}$ differs from the martingale measure $\bar{\mathcal{Q}}$. We call (λ_1, λ_2) the market price of risk.

Since the underlying securities $P(t, T)_{0 \leq t \leq T \leq \tau}$ are adapted to the filtration $\bar{\mathcal{F}}_{0 \leq t \leq \tau}$ not every $\mathcal{F}_\tau$-measurable contingent claim can be replicated with these zero-coupon bonds. Hence, the model is incomplete and there exist infinitely many equivalent martingale measures on $\mathcal{F}_\tau$. We use the concept of variance-minimizing martingale measures (see, for instance, Schweizer (1995)) for identifying a specific martingale measure $\mathcal{Q}$ and therefore obtaining unique prices of arbitrary contingent claims.[2] In Delbaen and Schachermayer (1996) variance-minimizing measures (relative to $\mathcal{P}$) are defined as those martingale measures which are closest to the given measure $\mathcal{P}$. It is quite intuitive that in our setting we have to extend $\bar{\lambda}$, $\bar{\eta}$ and $\bar{\mathcal{Q}}$ by defining the three-dimensional constant process

$$\lambda := (\lambda_1, \lambda_2, 0)$$

and the probability measure $\mathcal{Q}$ on $\mathcal{F}_{\mathcal{T}}$ by its density

$$\eta := \exp(\int_0^{\mathcal{T}} \lambda(t) dW(t) - \frac{1}{2} \int_0^{\mathcal{T}} |\lambda(t)|^2 dt)$$

with respect to the restriction of $\mathcal{P}$ to $\mathcal{F}_{\mathcal{T}}$.

Let X be a contingent claim which is paid at time T, i.e. a $\mathcal{F}_T$-measurable random variable. The price of X at time t is given by the conditional expectation

$$\mathbf{E}^{\mathcal{Q}}(X \cdot B(t)/B(T)|\mathcal{F}_t)$$

w.r.t. $\mathcal{Q}$.

In the following subsections we will specify models for market rates and deposit rates and volumes.

2.2 Market rates

We have implemented two specific classes of Gaussian HJM models: two-factor Vasicek models (Heath et al., 1992) and non-parametric models with piecewise constant volatility functions. Since we obtained better calibration results with the non-parametric models we will focus on this model class in the present paper.

2.2.1 Non-parametric HJM models

The existence of the unique martingale measure $\bar{\mathcal{Q}}$ implies that the forward rate drift under $\bar{\mathcal{P}}$ is uniquely determined by the volatility structure and the market price of risk:

$$\mu(t,T) = \sum_{k=1}^{2} \sigma_k(t,T)(\int_t^T \sigma_k(t,y)dy - \lambda_k). \tag{3}$$

Hence, it suffices to specify the volatility functions $\sigma(t,T) = (\sigma_1(t,T), \sigma_2(t,T))$. In the non-parametric model $\sigma_1(t,T)$ and $\sigma_2(t,T))$ are piecewise constant functions

[2] Intuitively, variance-minimizing measures compute the costs of variance-minimizing hedge strategies, i.e. strategies which minimize the variance of the replication error.

defined in the following way: let k be a natural number, $f_{11}, \dots, f_{1k+1}, f_{21}, \dots, f_{2k+1}$ and $m_0, m_1, \dots, m_{k+1}$ real numbers with

$$0 = m_0 < m_1 < \dots < m_{k+1} = \infty$$

and define for $j = 1, 2$ and $i = 1, \dots, k+1$

$$\sigma_j(t,T) \;:=\; f_{ji} \qquad \text{if } m_{i-1} \leq T - t < m_i.$$

In the current implementation we only support the following subdivision:[3]

$$k := 3, \quad m_1 := 1, \quad m_2 := 2, \quad m_3 := 5.$$

Hence, the volatility functions are fully specified by eight elements

$$f_{11}, \dots, f_{14}, f_{21}, \dots, f_{24}. \tag{4}$$

In our calibrations these eight volatility parameters provided sufficient flexibility to replicate the correlation structure of yield curves of different maturities rather well.

Price computations for arbitrary contingent claims will be based on short rate simulation. Therefore, we now evaluate the short rate formula (1), i.e.

$$r(t) = f(0,t) + \int_0^t \mu(v,t)dv + \int_0^t \sigma(v,t)d\bar{W}(v).$$

In the non-parametric model the integral $\int_0^t \mu(v,t)dv$ becomes a piecewise defined function. Each of the four pieces can be easily computed with *Mathematica*. For example:

$$\begin{aligned}
\int_0^t \mu(v,t)dv &= (1/2)\,(f_{11}^2 + f_{21}^2)\,t^2 - f_{11}\,t\,\lambda_1 - f_{21}\,t\,\lambda_2 & \text{if } 0 \leq t < 1,\\
&= -f_{11}\,f_{12} + f_{12}^2/2 + (1/2)\,(f_{11}^2 + f_{21}^2) - f_{21}\,f_{22} + f_{22}^2/2 & \\
&\quad + f_{11}\,f_{12}\,t - f_{12}^2\,t + f_{21}\,f_{22}\,t - f_{22}^2\,t + (1/2)\,(f_{12}^2 + f_{22}^2)\,t^2 & \\
&\quad + (-f_{11} + f_{12} - f_{12}\,t)\,\lambda_1 + (-f_{21} + f_{22} - f_{22}\,t)\,\lambda_2 & \text{if } 1 \leq t < 2,\\
&= \dots &
\end{aligned}$$

The integral $\int_0^t \sigma(v,t)d\bar{W}(v)$ is computed by Monte Carlo simulation. Hence, it is computationally easy to generate sample paths of the short rate although no closed form exists in the general case. Note, however, that the model can be specialized to the Ho-Lee short rate model (Ho and Lee, 1986) by setting

$$f_{11} = \dots = f_{14} := c \quad \text{and} \quad f_{21} = \dots = f_{24} := 0$$

with constant $c > 0$. It is well known that the short rate in the Ho-Lee model is normally distributed with mean $\to \infty$ if $t \to \infty$. The distribution of the short rate in the general non-parametric model has the same characteristics if f_{14} or f_{24} is unequal zero.

[3] For calibration we use LIBOR rates with maturities up to 1 year and swap rates with maturities of 2 years or more. Therefore, the interval $[0, 1)$ corresponds to the money market and the intervals $[2, 5)$ and $[5, \infty)$ to the swap market.

2.2.2 Parameter estimation in Gaussian HJM models

We assume that $\sigma(t,T)$ is a deterministic function which only depends on the time to maturity $T-t$, i.e. $\sigma(t_1,T_1)=\sigma(t_2,T_2)$ if $T_1-t_1=T_2-t_2$ for arbitrary t_1,t_2,T_1,T_2. Obviously, this condition is satisfied for the implemented model classes.

Let $r_i=(r_{i0},\dots,r_{im})$, $i=1,\dots,n$, be $m+1$ samples for n spot rates with maturities $\alpha_1,\dots,\alpha_n$. In this subsection we estimate the parameters in the volatility function $\sigma(t,T)$ by applying principal component analysis to the samples. Furthermore, we discuss two techniques for determining the market price of risk.

In a two-factor HJM model, the spot rates

$$R(t,T) := -\frac{log(P(t,T))}{T-t}$$

can be represented by

$$\begin{aligned} dR(t,T) &= -(1/(T-t))(r(t)-R(t,T)-\sum_{k=1}^{2}(\sigma_k^p(t,T)\lambda_k+(1/2)\sigma_k^p(t,T)^2))dt \\ &\quad -\sum_{k=1}^{2}(\sigma_k^p(t,T)/(T-t)dW_k(t), \end{aligned} \tag{5}$$

where

$$\sigma_k^p(t,T) \quad := \quad -\int_t^T \sigma_k(t,u)du \qquad k=1,2.$$

By discretization with step size Δ we obtain

$$\begin{aligned} R(t+\Delta,T) &= R(t,T)-(r(t)-R(t,T))\Delta/(T-t) \\ &\quad +\sum_{k=1}^{2}(\sigma_k^p(t,T)\lambda_k+(1/2)\sigma_k^p(t,T)^2)\Delta/(T-t) \\ &\quad -\sum_{k=1}^{2}\sigma_k^p(t,T)/(T-t)\Delta W_k(t), \end{aligned} \tag{6}$$

where $\Delta W_k(t)$ is a normally distributed variable with zero mean and variance Δ. Since $\sigma_1^p(t,T)$ and $\sigma_2^p(t,T)$ only depend on the time to maturity $T-t$ but not on t we define

$$\begin{aligned} \tilde{\mu}_k(\alpha) &:= (\sigma_k^p(t,t+\alpha)\lambda_k+(1/2)\sigma_k^p(t,t+\alpha)^2))\Delta/\alpha, \\ \tilde{\sigma}_k(\alpha) &:= -\sigma_k^p(t,t+\alpha)/\alpha \end{aligned}$$

for an arbitrary t. By (6),

$$A(t,\alpha)=\sum_{k=1}^{2}\tilde{\mu}_k(\alpha)+\sum_{k=1}^{2}\tilde{\sigma}_k(\alpha)\Delta W_k(t),$$

where

$$A(t,\alpha) := R(t+\Delta, t+\alpha) - R(t,t+\alpha) + (r(t) - R(t,t+\alpha))\Delta/\alpha. \tag{7}$$

In order to transform the spot rate samples $r_{i0}, \ldots, r_{im}$, $i = 1, \ldots, n$, into samples for the adjusted rates $A(t, \alpha_i)$ we take the spot rate of shortest maturity α_1 as proxy for the short rate. Furthermore, since no data is available for yields of maturity $\alpha_i - \Delta$ we use linear interpolation for computing $R(t+\Delta, t+\alpha_i)$: for $i = 2, \ldots, n$

$$\begin{aligned} A(t,\alpha_i) \;\approx\; & \frac{\Delta R(t+\Delta, t+\Delta+\alpha_{i-1}) + (\alpha_i - \alpha_{i-1} - \Delta) R(t+\Delta, t+\Delta+\alpha_i)}{\alpha_i - \alpha_{i-1}} \\ & - R(t, t+\alpha_i) + (R(t,t+\alpha_1) - R(t,t+\alpha_i))\Delta/\alpha_i. \end{aligned} \tag{8}$$

We use (8) for adjustment and center the adjusted samples. In this way we obtain m samples $s_i = (s_{i1}, \ldots, s_{im})$, $i = 2, \ldots, n$ for each of the centered and normally distributed variables

$$B(\alpha_i) := \sum_{k=1}^{2} \tilde{\sigma}_k(\alpha_i)\Delta W_k \qquad i = 2, \ldots, n. \tag{9}$$

Based on these samples we compute principal components

$$e_i := \sum_{j=1}^{n-1} p_{ij} s_{j+1} \qquad i = 1, \ldots, n-1 \tag{10}$$

with coefficient vectors $p_i = (p_{i1}, \ldots, p_{i,n-1})$. By definition of principal components, $e_1, \ldots, e_{n-1}$ are uncorrelated and have mean 0 and variances $\beta_1 > \ldots > \beta_{n-1}$. Furthermore,

$$s_{j+1} = \sum_{i=1}^{n-1} p_{ij} e_i.$$

For determining the volatility parameters we only consider the first two principal components:

$$s_{j+1} \;\approx\; p_{1j} e_1 + p_{2j} e_2 \qquad j = 1, \ldots, n-1.$$

We identify the normally distributed variable ΔW_k with the principle component e_k and choose the volatility parameters $\tilde{\sigma}_k(\alpha_{j+1})$ in such a way that variances match as well as possible: since

$$\mathrm{std}(\tilde{\sigma}_k(\alpha_{j+1})\Delta W_k) = \tilde{\sigma}_k(\alpha_{j+1})\sqrt{\Delta}, \qquad \mathrm{std}(p_{kj} e_k) = p_{kj}\sqrt{\beta_k},$$

we minimize the two functions

$$\sum_{j=1}^{n-1} (\tilde{\sigma}_k(\alpha_{j+1}) - p_{kj}\sqrt{\beta_k/\Delta})^2 \qquad k = 1, 2.$$

We now turn to the estimation of the two parameters λ_1 and λ_2 which determine the market price of risk. One straightforward approach is to derive the market price of risk from the mean of the spot rate samples: since the mean of $A(t,\alpha)$ is $\tilde{\mu_1}(\alpha)+\tilde{\mu_2}(\alpha)$ we minimize

$$\sum_{j=2}^{n}(\text{mean}(\tilde{s}_{j1},\ldots,\tilde{s}_{jm})-(\tilde{\mu_1}(\alpha_j)+\tilde{\mu_2}(\alpha_j)))^2,$$

where $\tilde{s}_{j1},\ldots,\tilde{s}_{jm}$ are samples of $A(t,\alpha_j)$. Our second estimation method is based on the minimization of the differences between the means of spot rates $R(t,t+\alpha_1),\ldots,R(t,t+\alpha_n)$ and estimates $E_1,\ldots,E_n$ for a particular time t.

We refer to Kalkbrener (2002) for a detailed presentation of calibrations of two-factor Vasicek and non-parametric HJM models.

2.3 Deposit rates

We model deposit rates by a deterministic function with only the market rates as stochastic arguments, no additional stochastic factor is used. Hence, every deposit rate $d(t)$ is a stochastic process adapted to the filtration $\bar{\mathcal{F}}_{0\leq t\leq\tau}$. In order to exclude arbitrage opportunities for individual investors we assume that $d(t)\leq r(t)$ for all t (see Jarrow and van Deventer (1998)).

The structure of the replicating portfolio heavily depends on the specific form of the deposit rate process. Therefore, special attention has to be given to the estimation of deposit rates. In our model, we do not impose any restrictions on the form of the deterministic function which specifies a deposit rate. We also allow market rates of different maturities as stochastic arguments. This increases the flexibility of the model. On the other hand, this freedom of choice does not give any guidance in determining a realistic deposit rate process. We think that the specification of deposit rates has to be based on a careful market analysis.

One possibility to specify deposit rates in our model is the following parametric family of processes suggested in Jarrow and van Deventer (1998):

$$d(t):=d(0)+\beta_0\,t+\beta_1\int_0^t r(s)ds+\beta_2\,(r(t)-r(0)).$$

The parameter estimates for β_0, β_1 and β_2 would reflect local market characteristics. Note that the discrete version of this process is similar to deposit rates used by O'Brien et al. (1994), the Office of Thrift Supervision (1994), Hutchinson and Pennacchi (1996) and Selvaggio (1996).

2.4 Deposit volumes

The deposit volume $V(t)$ is a stochastic process adapted to the filtration $\mathcal{F}_{0\leq t\leq\tau}$. We have currently implemented two different volume models: a normally and a lognormally distributed model. In the normal model, $V(t)$ is defined as the sum of a deterministic linear function $f(t) = a + b \cdot t$ and an Ornstein-Uhlenbeck process $X(t)$, i.e.

$$V(t) = f(t) + X(t), \tag{11}$$

where

$$dX(t) = \mu_V X(t)dt + \sigma_V d\tilde{W}(t)$$

with constants $\sigma_V > 0, \mu_V$ and Brownian motion $\tilde{W}(t)$ (under $\mathcal{P}$). Hence, the function $f(t) = a + b \cdot t$ specifies the trend of the volume $V(t)$. The Ornstein-Uhlenbeck process $X(t)$ is mean reverting and therefore the volume fluctuates around the linear trend with mean reversion μ_V and volatility σ_V.[4] We allow constant correlations c_1, c_2 between $\tilde{W}$ and the two Brownian motions W_1, W_2 in the HJM model. Hence, we set

$$\tilde{W}(t) := c_1 W_1(t) + c_2 W_2(t) + \sqrt{1 - c_1^2 - c_2^2}\, W_3(t). \tag{12}$$

The parameters a, b, μ_V, σ_V are estimated from a historical time series of deposit volumes $v_0, \dots, v_m$. We use linear regression to determine the parameters a and b in the time-dependent trend function $f(t)$. For estimating μ_V and σ_V we detrend the historical volumes by subtracting $f(t)$, i.e.

$$\bar{v}_i := v_i - (a + (i - m)\Delta b),$$

where Δ denotes the length of the time interval between two observations v_i and v_{i+1}. In order to be consistent with the interest rate model we discretize the Ornstein-Uhlenbeck process $X(t)$ for estimating μ_V, σ_V:

$$X(t + \Delta) = X(t) + \mu_V X(t)\Delta + \sigma_V \Delta\tilde{W}(t).$$

Therefore, by using adjustment

$$X(t + \Delta) - X(t) - \mu_V X(t)\Delta$$

we transform the detrended deposit volumes $\bar{v}_0, \dots, \bar{v}_m$ into samples $u = (u_1, \dots, u_m)$ for a centered, normally distributed variable with variance $\sigma_V^2\Delta$. We choose μ_V and σ_V to match the first two moments. Alternatively, μ_V and σ_V could be estimated without discretization by a maximum-likelihood method (which is equivalent to least squares for OU-processes).

It remains to estimate the correlation parameters c_1 and c_2. Let e_1, e_2 be the first two principal components defined in (10). Since we have identified e_1 (resp. e_2) with

[4]Note that the deposit volume $V(t)$ may become negative. This is an unpleasant feature of the normal model which could cause problems for certain types of deposit accounts (although it never did in our case studies). One possible remedy is to floor $V(t)$ at 0.

ΔW_1 (resp. ΔW_2) we obtain c_1 (resp. c_2) as correlation between u and e_1 (resp. e_2).

The lognormal model is specified rather similarly: the definition (11) of $V(t)$ is replaced by

$$V(t) = e^{f(t)+X(t)},$$

the function f and the process X are defined as above. The same procedures can be used for parameter calibration in the lognormal model, the only difference is that historical volumes have to be replaced by their logarithms.

3 Liquidity risk management

Liquidity risk management is based on the concept of the term structure of liquidity. Let $0 \leq t \leq \tau$ and define the process of minima by

$$M(t) := \min_{0 \leq s \leq t} V(s).$$

In each scenario the stochastic process $M(t)$ specifies the minimal volume in $[0, t]$. This is the exact amount which is available for investment over the entire period $[0, t]$. For $0 < p < 1$ denote the p-quantile of $M(t)$ by $TSL(t, p)$. Obviously, the function TSL maps $[0, \tau] \times (0, 1)$ to the real numbers. It is called term structure of liquidity.

The probability that the volume V drops below level $TSL(t, p)$ in the time interval $[0, t]$ is exactly p. In other words, $TSL(t, p)$ is the exact amount which is available for investment over the entire period $[0, t]$ with probability p. The term structure of liquidity can therefore be used to compute liquidity restrictions and to immunize a replicating portfolio against liquidity risk.

The term structure of liquidity is computed by Monte Carlo simulation: since

$$\begin{aligned} M(t) &= \min_{0 \leq s \leq t} V(s), \\ V(t) &= a + bt + X(t), \\ dX(t) &= \mu_V X(t) dt + \sigma_V d\tilde{W}(t) \end{aligned}$$

(under $\mathcal{P}$) we can compute several paths of $V(t)$ using standard simulation techniques for diffusion processes. For each path of $V(t)$ we construct the corresponding minimum path of $M(t)$. The quantile $TSL(t, p)$ is determined by ordering the values of the minimum paths at t and choosing the appropriate element in the ordered list.

4 Interest rate risk management

Our main objective is the construction of a replicating bond portfolio with fixed maturities. This bond portfolio is derived from the delta profile of the non-maturing liabilities. For computing the delta profile a pricing methodology is needed.

4.1 Pricing of non-maturing liabilities

Since deposit rates are typically lower than market rates on comparable (equal risk) financial securities these deposits have a positive value for financial institutions. We use Monte Carlo simulation to determine this value. First the time interval $[0, \tau]$ is discretized with step size Δ and trading dates $0 = t_0, t_1, \ldots, t_{m-1}, t_m = \tau$ with $t_i - t_{i-1} = \Delta$. The cash flow $C_V(t)$ of the demand deposits at time t_i is defined by

$$\begin{aligned} &C_V(t_0) := V(t_0) \\ &C_V(t_i) := -V(t_{i-1})e^{d(t_{i-1})\Delta-1} + V(t_i) - V(t_{i-1}) \quad \text{for } i = 1, \ldots, m-1 \\ &C_V(t_m) := -V(t_{m-1})e^{d(t_{m-1})\Delta-1} - V(t_{m-1}) \end{aligned}$$

The net present value of the demand deposits at time 0 is given by

$$V(t_0) + \mathbf{E}^{\mathcal{Q}}(\sum_{i=1}^{m-1} C_V(t_i)/B(t_i)) + \mathbf{E}^{\mathcal{Q}}(C_V(t_m)/B(t_m)). \tag{13}$$

The price formula (13) can be transformed into the following equivalent expression (see Jarrow and van Deventer (1998))

$$\mathbf{E}^{\mathcal{Q}}(\sum_{i=0}^{m-1} V(t_i)(r(t_i) - d(t_i))/B(t_{i+1})). \tag{14}$$

Hence, the net present value can be interpreted as the value of an exotic interest rate swap paying floating at $d(t)$ and receiving floating at $r(t)$ on a random principal of $V(t)$.

For computing (14) we simulate the following processes [5] (under $\mathcal{Q}$):

$$\begin{aligned} r(t) &= f(0,t) + \int_0^t \sum_{k=1}^{2} \sigma_k(v,t)(\int_v^t \sigma_k(v,y)dy - \lambda_k)dv + \int_0^t \sigma(v,t)d\bar{W}(v), \\ d(t) &= \text{deposit rate given by a deterministic function of the short rate,} \qquad (15) \\ B(t) &= \exp(\int_0^t r(v)dv), \\ V(t) &= a + bt + X(t), \end{aligned}$$

where $X(t)$ is defined by

$$dX(t) = -(c_1\lambda_1 + c_2\lambda_2)\sigma_V + \mu_V X(t)dt + \sigma_V d\tilde{W}(t)$$

and $\tilde{W}(t)$ is a Brownian motion correlated to W_1 and W_2 with correlation coefficients c_1 and c_2.

[5] Additionally, spot rates have to be simulated if they are used as stochastic arguments for the deposit rate. We refer to Kalkbrener (2002) for the simulation of spot rates in two-factor Vasicek and non-parametric HJM models.

4.2 Construction of the replicating portfolio

As soon as a pricing methodology is available the computation of sensitivities to the different input parameters is straightforward. In particular, deltas can be computed by shifting the yield curve resp. parts of the yield curve. The replicating bond portfolio is constructed in such a way that it has the same price and the same delta profile as the non-maturing liabilities.

Before we can apply the replication methodology we have to modify the specification of the volume process. Note that $V(t)$ specifies future volumes of deposits including increases in volume due to new deposits in the period $[0, t]$. In contrast, the purpose of the bond portfolio constructed at time 0 is to replicate or hedge the current deposit volume. Future increases are not hedged at time 0 but as soon as the new deposits are available. As a consequence the construction of the replicating portfolio has to be based on a process which reflects the stability of the current volume but ignores future volume increases. The natural choice is the process

$$M(t) = \min_{0 \leq s \leq t} V(s)$$

which specifies the volume which is available for investment over the entire period $[0, t]$. This process has already been used in the construction of the term structure of liquidity. Note that $M(t)$ is dominated by $V(t)$ and non-increasing, i.e. $M(t) \leq V(t)$ for every t and $M(s) \geq M(t)$ for $s \leq t$.[6]

The construction of the replicating portfolio can now be summarized in the following way:

1. The price and deltas of the non-maturing liabilities are computed with valuation formula (14), where $V(t)$ is replaced by $M(t)$ in (14) and (15).

2. The price and delta profile are then replicated by a bond portfolio.

Our methodology can be easily extended to replicating portfolios which contain not only bonds. For instance, if certain deposits show significant vega risk caps and floors could be included (Ho, 1992; Ho and Chen, 1995).

5 Case study: risk management of savings accounts

The objective of this section is to demonstrate the methodology by means of a concrete case study: the risk management of savings accounts.

5.1 Liquidity risk management

Consider the following time series of daily volumes of savings accounts from Nov. 1996 to May 2000 (in 100 million EUR):

[6] $M(t)$ is maximal with this property: if a process $\bar{M}(t)$ satisfies $\bar{M}(t) \leq V(t)$ for every t and $\bar{M}(s) \geq \bar{M}(t)$ for $s \leq t$ then $\bar{M}(t) \leq M(t)$ for every t.

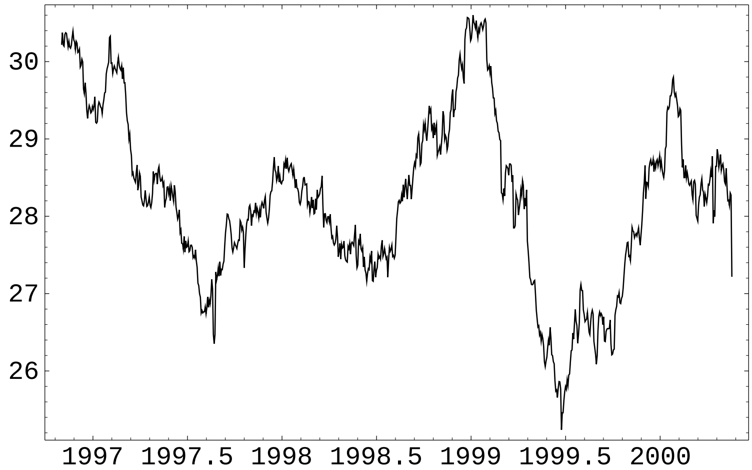

Figure 1: Volumes of savings accounts Nov. 96 to May 00

We assume normal distribution of volume increments and model the volume process $V(t)$ as the sum of a deterministic linear function and an Ornstein-Uhlenbeck process (see equation (11))[7], i.e.

$$V(t) = f(t) + X(t), \tag{16}$$

where

$$f(t) = a + b \cdot t \quad \text{and} \quad dX(t) = \mu_V X(t)dt + \sigma_V d\tilde{W}(t).$$

The parameters a and b are determined by linear regression. A maximum-likelihood estimator is applied to the detrended time series for computing μ_V and σ_V:

$$a = 27.72, \quad b = -0.28, \quad \mu_V = -2.64, \quad \sigma_V = 2.34.$$

We now compute the term structure of liquidity $TSL(t,p)$ for a period of 10 years starting May 2000 and quantiles $p = 0.1\%,\ 1\%,\ 5\%$ (see figure 2).

Based on the 1%-quantile a bucketing of the current volume of 2.72bn EUR is obtained.

Liquidity bucketing (in 100mn EUR):

ON	3m	6m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
2.1	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	21.5

[7]In this example the incorporation of a seasonal component might improve the quality of the forecast.

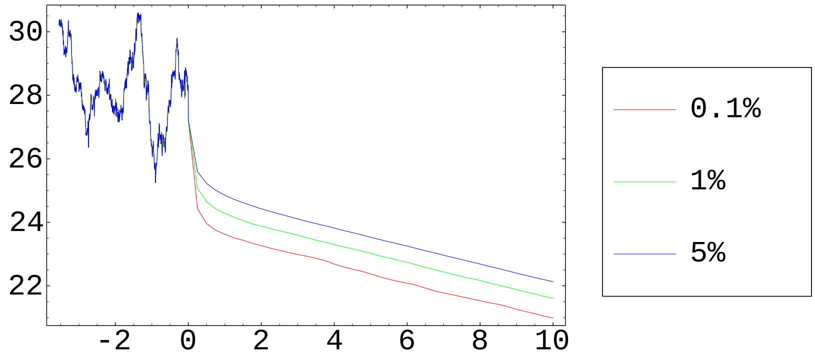

Figure 2: Quantiles May 00 to May 10

Note that the only reason for the dominance of the 10y bucket is the artificial assumption that all the money is paid back after 10 years. A longer planning period could be considered instead.

The estimated trend $b = -0.28$ has a significant impact on the liquidity buckets. As a general rule it is recommended not to rely entirely on estimates based on historical data but to consider alternative scenarios as well.[8] The quantitative framework presented in this paper facilitates this type of analysis. We now repeat the computation of the term structure of liquidity but use $b = 0.2$ instead, i.e. the average increase in volume is 20mn EUR per year. As above the liquidity buckets are based on the 1%-quantile.

Liquidity bucketing (based on average increase of 20mn EUR per year):

ON	3m	6m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
2.1	0.4	0.2	0.1	0.06	0.03	0.01	0	0	0	0	0	24.3

The ON and 3m buckets are unchanged because they are dominated by the volatility of the volumes. However, the buckets between 6m and 9y are significantly reduced and the money is shifted to the 10y bucket.

5.2 Interest rate risk management

For specifying the interest rate model we first derive 13 zero rates with maturities 1m, 3m, 6m, 1y, 2y,...,10y from daily time series of German 1m, 3m, 6m, 1y Libor and 2y, 3y, 5y, 7y, 10y swap rates[9]. The time period is from 3 April 1987 to 18 May 2000. The following chart shows the zero rates with maturities 1m, 6m, 2y and 10y.

[8]The analysis of alternative scenarios is particularly important if historical data is only available for a relatively short period of time (as the volume data in this case study).

[9]Data source: Datastream

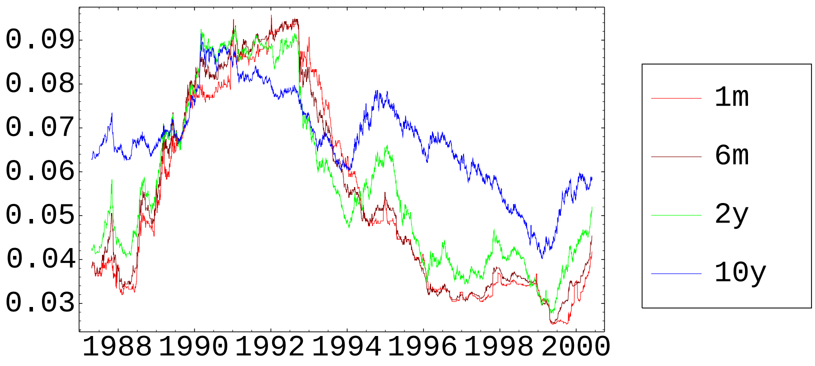

Figure 3: German zero rates April 87 to May 00

Interest rates are modeled by the non-parametric two-factor HJM model introduced in subsection 2.2. The calibration is based on principal component analysis. As usual, the first component corresponds to a positive shift of the entire yield curve[10] and the second component is downward sloping and causes the term structure to tilt (see, for instance, James and Webber (2000)). From theses two principal components the calibration procedure derives the following values for the eight volatility parameters in (4) and the two market price of risk parameters:

$$f_{11} = 0.0032 \quad f_{12} = 0.0105 \quad f_{13} = 0.0074 \quad f_{14} = 0.0057$$
$$f_{21} = 0.0058 \quad f_{22} = -0.0041 \quad f_{23} = -0.0016 \quad f_{24} = -0.0033$$

$$\lambda_1 = 0.3784 \quad \lambda_2 = 0.8755$$

Together with the zero rate curve on 18 May 2000

1m	3m	6m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
4.18	4.38	4.54	4.86	5.2	5.34	5.45	5.55	5.63	5.72	5.76	5.82	5.87

these parameters uniquely determine the market rate model for our 10y planning period. In order to ensure realistic rates over longer periods (see subsection 2.2.1) either market rates with maturities $> 10y$ or extrapolation techniques should be used in the calibration procedure.

The next task is the specification of deposit rates. Figure 4 shows the 1m and 10y zero rates together with the average basic rate[11] on savings accounts in Germany. In

[10] It is interesting that the first principal component of the German interest rates is not flat: we observed a stronger shift in the swap market than in the money market.

[11] Data source: Deutsche Bundesbank; monthly time series SU0022 for average basic rate on German savings accounts with three months notification period from April 1987 to May 2000.

this case study, the average rate is used as proxy for the specific deposit rate paid on the savings accounts. If sufficient internal data is available the internal time series should be used instead.

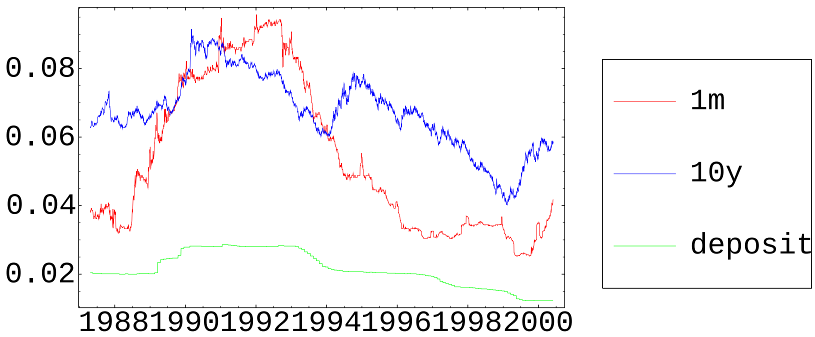

Figure 4: Deposit rate April 87 to May 00

In this analysis we model deposit rates as a piecewise linear function $d(x)$ of the short rate $r(t)$, i.e. deposit rates are specified as stochastic process $d(r(t))$ with

$$\begin{array}{lcll} d(x) := & x & \text{if} & x \leq 0 \\ d(x) := & \frac{(0.01+0.2\cdot 0.035)x}{0.035} & \text{if} & 0 < x \leq 0.035 \\ d(x) := & 0.01 + 0.2 \cdot x & \text{if} & 0.035 < x \end{array}$$

Hence, if the short rate is above 3.5% the savings rate is defined as 20% short rate plus a spread of 1%. The definition of the savings rate in the two other cases, i.e. negative short rate resp. short rate between 0% and 3.5%, ensures that the savings rate is continuous and not higher than the short rate. The actual numbers in the definition of $d(x)$ have been obtained by calibration.

The estimate in figure 5 has been computed by applying the function $d(x)$ to the time series of the 1m zero rate which is used as proxy for the short rate.

The chart shows that this simple function gives a good fit for the entire period with the exception of the last 8 months: market rates have risen substantially from October 1999 to May 2000 but the basic savings rate stayed at 125bp. One possible solution to this problem is the use of a more sophisticated function $d(x)$, for instance based on a delayed or asymmetric response of deposit rates.

It remains to determine correlations between volumes and market rates. Since the internal time series of volumes covers only 3.5 years we use Bundesbank data instead. Based on a monthly time series of the total volume of German savings

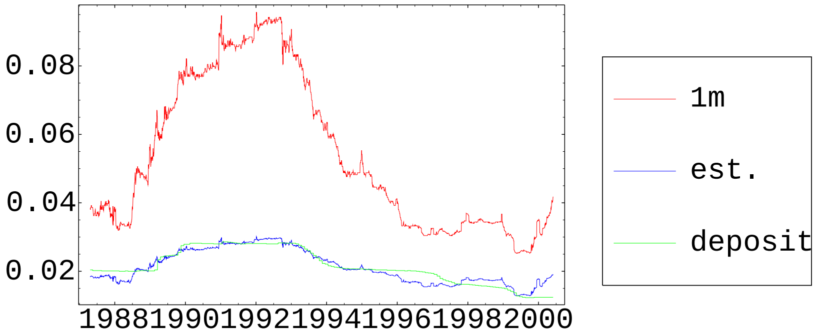

Figure 5: Estimated deposit rate

accounts for the period from 1980 to 2000 we obtain the following values for the correlation parameters c_1 and c_2 in (12):

$$c_1 = -0.07 \quad \text{and} \quad c_2 = 0.21.$$

The second stochastic factor in the market rates model corresponds to an increase of the short end and a decrease of the long end of the yield curve (see estimates for parameters $f_{21}, f_{22}, f_{23}, f_{24}$). Therefore, a positive correlation c_2 of 21% is not surprising.

The model is now completely specified and can be used for pricing and hedging. Applying the pricing methodology introduced in section 4 we obtain the value of savings accounts (without new business) for a 10 year period.

Value of savings accounts: 674mn EUR

In more detail, the value 674mn EUR of savings accounts is the difference of the current balance of 2.721bn EUR and the value 2.047bn EUR of all payments to customers. These payments are interest rate payments, withdrawls and repayments in 10 years. The value is mainly influenced by the margin between deposit rate and market rates, by the stability of volumes and the planning horizon.

The replicating portfolio is constructed in such a way that it has the same value of 2.047bn EUR and the same delta profile.

Replicating portfolio (in 100mn EUR):

ON	3m	6m	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
4.7	0.5	0.8	0.8	1.1	0.6	0.4	0.5	0.4	0.5	0.2	0.3	9.7

As already discussed in the section on liquidity risk management the reason for the dominance of the 10y bucket is the assumption that all the money is paid back after 10 years. Apart from the planning horizon sensitivities of the deposit rate to market rates and stability of volumes are the parameters which mainly determine the structure of the replicating portfolio. For instance, an increased sensitivity to the short rate or rapidly decreasing volumes shift money from the long end to the short end of the replicating portfolio.

6 Summary and future research

In this paper we have presented a general quantitative framework for liquidity risk and interest rate risk management for non-maturing liabilities.

Our approach to liquidity risk management is rather straightforward: we derive a stochastic process for deposit volumes from a historical time series and compute the term structure of liquidity. This term structure gives the probabilities that the volume drops below specified levels in specified time intervals and is used to immunize a replicating portfolio against liquidity risk. Deposit volumes are either defined as Ornstein-Uhlenbeck processes with linear trend or exponential functions of those processes. Despite its simplicity the model produced reliable results for the deposit volumes we worked with. Nevertheless it seems worthwhile to experiment with more realistic volume models, for instance models based on extreme value theory (Embrechts et al., 1997).

Another approach to modeling deposit volumes is to investigate correlations to macro economic variables. The analysis of the dependence of customer behaviour on the macro economic environment can certainly give important insights. However, it seems questionable whether more reliable volume forecasts can be obtained in this way since forecasting macro economic developments over longer periods is not a trivial task.

The interest rate risk management techniques we propose are based on arbitrage-free valuation. This methodology is transparent and consistent with the current practice of derivatives pricing and with the risk management methods for the trading book. Furthermore, our portfolio replication methodology is in line with suggestions made by regulatory authorities.

In this paper we have focused on the construction of replicating bond portfolios. However, the methodology can be easily extended to portfolios which contain not only bonds but also derivatives like caps and floors. Further analysis and comparison to the techniques suggested by Thomas Ho (Ho, 1992; Ho and Chen, 1995) are important topics for future research.

An interesting alternative to arbitrage-free pricing are risk management techniques based on stochastic optimization. These techniques seem to be a natural approach to a number of financial planning problems, for instance to investment problems in asset and liability management. Stochastic optimization models not only reflect the uncertainty in the future development of risk factors. They also provide a framework for modeling different investment strategies over the entire planning period and techniques for determining the optimal solution. The main obstacle in applying stochastic optimization to realistic investment problems is the complexity of this approach. In general, large optimization problems have to be solved if risk factors and cash flows are modeled in a realistic way. Recently, stochastic optimization techniques have been applied to margin optimization for non-maturing liabilities (Schürle, 1998; Frauendorfer and Schürle, 2000). It seems promising to adapt these techniques to the portfolio replication problem considered in this paper.

Our model for interest rate risk management of non-maturing liabilities consists of three building blocks: market rates, deposit rates and deposit volumes. Market rates are specified by a two-factor Vasicek or non-parametric HJM model. In our tests the non-parametric model outperformed the Vasicek model and met the requirement of realistic interest rates over long time periods rather well (Kalkbrener, 2002). The disadvantage of the non-parametric model is its complexity: since the volatility functions do not have analytic form simulation of interest rate paths is time-consuming. A comparison to affine models (Brown and Schaefer, 1994; Duffie and Kan, 1996; Dai and Singleton, 1998) is planned.

The value of deposit accounts and the structure of the replicating portfolio heavily depend on the specific form of the deposit rate process. In our model, no restrictions are imposed on the form of the deterministic function which specifies a deposit rate. We also allow market rates of different maturities as stochastic arguments. Our objective is to provide a framework which is flexible enough for modeling different types of deposit rates. The actual process definitions for specific types of deposit rates should to be based on a careful market analysis: problems like the dependence on market rates of different maturities or the asymmetric response to market rate changes, i.e. whether deposit rates move more rapidly when market rates drop than when they rise, deserve a closer study. This type of analysis is beyond the scope of the paper.

References

[1] L. Ausubel (1991). The failure of competition in the credit card market. *American Economic Review* 81 (1), 50-81.

[2] Basel Committee on Banking Supervision (1999). A new capital adequacy framework. Consultative paper, Basel, Switzerland.

[3] R.H. Brown and S.M. Schaefer (1994). Interest rate volatility and the shape of the term structure. *Philosophical Transactions of the Royal Society of London A* 347, 563-576.

[4] Q. Dai and K.J. Singleton (1998). Specification analysis of affine term structure models. *Journal of Finance* LV, 1943-1978.

[5] F. Delbaen and W. Schachermayer (1996). The variance-optimal martingale measure for continuous processes. *Bernoulli* 2, 85-105.

[6] D. Duffie and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance* 6, 379-406.

[7] P. Embrechts, C. Klüppelberg and T. Mikosch (1997). *Modelling extremal events.* Springer, Berlin Heidelberg, Germany.

[8] European Commission, Internal Market Directorate General (1999). A review of regulatory capital requirements for EU credit institutions and investment firms. Consultation document, MARKT/1123/99-EN-Rev.1, Brussels, Belgium.

[9] Federal Reserve (1995). Risk-Based Capital Standards: Interest Rate Risk. Docket No. R-0802, Washington, D.C., USA.

[10] K. Frauendorfer and M. Schürle (2000). Stochastic optimization in asset and liability management: a model for non-maturing accounts. In: S.P. Uryasev (ed.), *Probabilistic Constrained Optimization*, Kluwer Academic Publishers, Dordrecht, The Netherlands.

[11] D. Heath, R. Jarrow and A. Morton (1992). Bond pricing and the term structure of interest rates: a new methodology for contingent claim valuation. *Econometrica* 60 (1), 77-105.

[12] F. Heitmann and S. Trautmann (1995). Gaussian multi-factor interest rate models: theory, estimation, and implications for option pricing. Unpublished manuscript, Johannes Gutenberg-University Mainz, Germany.

[13] T. Ho (1992). Managing illiquid bonds and the linear path space. *Journal of Fixed Income* 2 (1).

[14] T. Ho and M. Chen (1995). Arbitrage-free bond canonical decomposition. Global Advanced Technology Corporation, New York, USA.

[15] T. Ho and S. Lee (1986). Term structure movements and pricing interest rate contingent claims. *Journal of Finance* 41, 1011-1029.

[16] D. Hutchinson and G. Pennacchi (1996). Measuring rents and interest rate risk in imperfect financial markets: the case of retail bank deposits. *Journal of Financial and Quantitative Analysis* 31, 399-417.

[17] J. James and N. Webber (2000). *Interest Rate Modelling.* Wiley, Chichester, UK.

[18] T. Janosi, R. Jarrow and F. Zullo (1999). An empirical analysis of the Jarrow-van Deventer model for valuing non-maturity deposits. *Journal of Derivatives* 7 (Fall), 8-31.

[19] R. Jarrow and D. van Deventer (1998). The arbitrage-free valuation and hedging of demand deposits and credit card loans. *Journal of Banking and Finance* 22, 249-272.

[20] Kalkbrener (2002). Calibration and simulation of Gaussian two-factor HJM models. In preparation.

[21] J. O'Brien, A. Orphanides and D. Small (1994). Estimating the interest rate sensitivity of liquid retail deposit values. In: *Proceedings of the Conference on Bank Structure and Competition.*

[22] J. O'Brien (2000). Estimating the value and interest rate risk of interest-bearing transactions deposits. Division of Research and Statistics, Board of Governors of the Federal Reserve System, Washington, D.C., USA.

[23] Office of Thrift Supervision, Department of the Treasury (1994). The OTS Net Portfolio Value Model.

[24] M. Schürle (1998). *Zinsmodelle in der stochastischen Optimierung.* Volume 279 in Bank - und finanzwirtschaftliche Forschungen, Paul Haupt, Berne, Switzerland.

[25] M. Schweizer (1995). Variance-optimal hedging in discrete-time. *Mathematics of Operations Research* 20, 1-32.

[26] R. Selvaggio (1996). Using the OAS methodology to value and hedge commercial bank retail demand deposit premiums. In: F. Fabozzi and A. Konishi (Eds.), *The Handbook of Asset/Liability Management.*

[27] S. Wolfram (1991). *Mathematica: a System for Doing Mathematics by Computer.* Addison-Wesley, Reading, MA.